Appendix

This Appendix includes additional material that provides background for some of the arguments made in the paper. Section A.1 provides an empirical analysis of the relationship between domestic debt and inequality. Section A.2 presents a deterministic version of our model without default risk and log preferences. Section A.3 extends this simplified model to incorporate aggregate risk. Section A.4 derives analytically the equilibrium price function for this model and describes its determinants. Section A.5 provides the details of the calibration exercise presented in the paper. Section A.6 displays a set of additional figures of the benchmark model. Section A.7 contains a sensitivity analysis of the benchmark model. Sections A.8 and A.9 introduce additional figures for the extension of the model with biased welfare weights and foreign investors, respectively. Finally, Section A.10 describes the extension of the model with two assets in detail and presents a set of figures that complement those in the main body of the paper.

A.1. Empirical Analysis Debt and Inequality

Further and more systematic empirical analysis of the relationship between wealth inequality and public debt shows a statistically significant, bell-shaped relationship: Debt is increasing in wealth inequality when inequality is low, until the Gini coefficient reaches about 0.75, and then becomes decreasing in wealth inequality. This finding follows from estimating a cross-country panel regression using the international database of Gini coefficients for wealth produced by Davies et al. (2009) and Davies et al. (2012), together with data on public debt-output ratios and control variables (i.e. the size of the government, proxied by the ratio of government expenditures to GDP, and country fixed effects). The results of panel regressions of the debt ratio on the Gini

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coefficient, the square of this coefficient, and the controls (using the observations for 2000 and 2012) are shown in Table A.1, and Figure A.1 shows a plot of the 2012 data and the fitted regression curve. The regressions only explain about a tenth of the cross-country variations on debt ratios, but the linear and quadratic effects of the wealth Gini coefficient are statistically significant and opposite in sign.

![Figure A.1. Public debt ratios and wealth inequality. Author’s calculations based on WDI, Davies et al. (2009) and Davies et al. (2012). Fitted line from results presented in Table A.1.](image)

**Table A.1.** Panel regressions of public debt ratios and wealth inequality.

<table>
<thead>
<tr>
<th></th>
<th>Panel Country Fixed Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dep. Var: Debt to GDP</td>
</tr>
<tr>
<td>Wealth Gini</td>
<td>17.65**</td>
</tr>
<tr>
<td>s.e.</td>
<td>7.06</td>
</tr>
<tr>
<td>Wealth Gini²</td>
<td>-11.80**</td>
</tr>
<tr>
<td>s.e.</td>
<td>4.76</td>
</tr>
<tr>
<td>G/Y</td>
<td>-0.010</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.012</td>
</tr>
<tr>
<td>avg. country FE</td>
<td>-5.92**</td>
</tr>
<tr>
<td>s.e.</td>
<td>2.60</td>
</tr>
<tr>
<td>years</td>
<td>2000-2012</td>
</tr>
<tr>
<td>R²</td>
<td>0.118</td>
</tr>
<tr>
<td># obs.</td>
<td>139</td>
</tr>
</tbody>
</table>

Notes: Author’s calculations based on WDI, Davies et al. (2009) and Davies et al. (2012). Debt to GDP refers to Central Government Debt as share of GDP (source WDI). Wealth Gini as reported for each country for years 2000 and 2012 by Davies et al. (2009) and Davies et al. (2012). G/Y corresponds to government final consumption as a share of GDP (source WDI). Coefficients and standard errors reported are derived from a panel linear regression with Country Fixed Effects. The sample contains 87 countries with data for years 2000 and 2012. If debt to GDP or government expenditure ratio is not available for year 2000 or 2012 data for year 1999 or 2011 is used.
A.2. Deterministic Model without Default Risk and Log Utility

This part of the Appendix derives solutions for a version of the model in which low-wealth (L) types do not hold any bonds and high-wealth (H) types buy all the debt. We cover first the fully deterministic case, without any shocks to income or government policies, and no default risk, but government expenditures may be deterministically different across periods. Government wants to use debt to relocate consumption across agents and across periods optimally given a utilitarian welfare function. Ruling out default on initial outstanding debt, the planner trades off the desire to use debt to smooth taxation for L types (reduce date-0 taxes by issuing debt) against the cost of the postponement of consumption this induces on H types who save to buy the debt. In what follows, we work with cases where $b^L_1 = 0$ which might not hold depending on the parameters of the model; however, it is a relevant benchmark since this holds for our baseline parameterization and all the extensions of the model we present. This assumption together with log utility provides closed form solutions. The goal is to illustrate the mechanisms that are driving the model when default risk and stochastic government purchases are taken out. Later in the Appendix we derive some results for the model with stochastic government purchases, and make some inferences for the case with default risk.

A.2.1. Households. A fraction $\gamma$ of agents are L types, and $1 - \gamma$ are H types. Preferences are:

$$\ln(c_0^i) + \beta \ln(c_1^i) \quad \text{for} \quad i = L, H \quad (A.1)$$

If $b^L_1 = 0$, budget constraints are:

$$c_0^L = y - \tau_0, \quad c_0^H = y - \tau_0 + b^H_0 - qb^H_1$$
$$c_1^L = y - \tau_1, \quad c_1^H = y - \tau_1 + b^H_1 \quad (A.2)$$

Since L types do not save they can only consume what their budget constraints allow. This is important because altering taxes affects disposable income, which will in turn affect the optimal debt choice of the government. For H types, the Euler equation is:

$$q = \frac{\beta c_0^H}{c_1^H} \quad (A.4)$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, it must be the case that they are credit constrained (i.e. they would want to hold negative assets). That is, at the equilibrium price of debt their Euler equation for bonds would satisfy:

$$q > \frac{\beta c_0^L}{c_1^L} \quad (A.5)$$

1. The $L-$ type agents will be credit constrained (i.e. $b^L_1 = 0$) the lower $b^H_0$, $B_0$ and $B_1$. 

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A.2.2. Government. The government budget constraints are:

\[ \tau_0 = g_0 + B_0 - qB_1 \]  
\[ \tau_1 = g_1 + B_1 \]  

(A.6)  
(A.7)

The initial debt \( B_0 \geq 0 \) is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

\[ \gamma (\ln(c_0^L) + \beta \ln(c_1^L)) + (1 - \gamma)(\ln(c_0^H) + \beta \ln(c_1^H)) \]  

(A.8)

A.2.3. Competitive Equilibrium in the Bond Market. A competitive equilibrium in the bond market for a given supply of government debt \( B_1 \) is given by a price \( q \) that satisfies the market-clearing condition of the bond market: \( \gamma b_1^L + (1 - \gamma)b_1^H = B_1 \). By construction the same condition is assumed to hold for the initial conditions \( b_0^H = B_0/(1 - \gamma) \).

Rewriting the Euler equation of H types using the budget constraint, the government budget constraints and the bond market-clearing conditions when \( b_1^L = 0 \) yields:

\[ q = \beta \frac{y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0 - q \left( \frac{\gamma}{1 - \gamma} \right) B_1}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) B_1} \]  

(A.9)

Hence, the equilibrium price of bonds for a given government supply is:

\[ q(B_1) = \beta \frac{y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) (1 + \beta) B_1} \]  

(A.10)

Note that this price is not restricted to be lower than 1 (i.e. \( q(B_1) > 1 \) which implies a negative real rate of return on government debt can be an equilibrium outcome). In particular, as \( \gamma \) rises the per capita bond demand of H-types increases and this puts upward pressure on bond prices, and even more so if the government finds it optimal to offer less debt than the initial debt, as we showed numerically and explain further below. As \( \gamma \to 1 \), the limit of the equilibrium price goes to \( q(B_1) = \frac{\beta}{1 + \beta} \frac{B_0}{B_1} \) even though market-clearing requires the demand of the infinitesimal small H type to rise to infinity.

After some simplification, the derivative of this price is given by:

\[ q'(B_1) = \frac{-q(B_1) \left( \frac{\gamma}{1 - \gamma} \right) (1 + \beta)}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) (1 + \beta) B_1} \]  

(A.11)

which at any equilibrium with a positive bond price satisfies \( q'(B_1) < 0 \) (notice \( c_1^H > 0 \) implies that the denominator of this expression must be positive).
Consider now what happens to this equilibrium as the fraction of L-types vanishes. As \( \gamma \to 0 \), the economy converges to a case where there is only an H type representative agent, and the price is simply \( q(B_1) = \beta(y - g_0)/(y - g_1) \), which is in fact independent of \( B_1 \) and reduces to \( \beta \) if government purchases are stationary. Trivially, in this case the planner solves the same problem as the representative agent and the equilibrium is efficient. Also, for an exogenously given \( B_0 \) and stationary \( g \), the competitive equilibrium is stationary at this consumption level:

\[
c^H = y - g + \left( \frac{\gamma}{1 - \gamma} \right) \frac{B_0}{1 + \beta}
\]

and the optimal debt is:

\[
B_1 = \frac{B_0}{1 + \beta}
\]

Hence, in this case consumption and disposable income each period are fully stationary, yet the optimal debt policy is always to reduce the initial debt by the fraction \( 1/(1 + \beta) \). This is only because of the two-period nature of the model. With an infinite horizon, the same bond price would imply that an equilibrium with stationary consumption and an optimal policy that is simply \( B_1 = B_0 \). It also follows trivially that carrying no initial debt to start with would be first-best, using lump-sum taxation to pay for \( g \).

### A.2.4. Optimal Debt Choice.

The government’s optimal choice of \( B_1 \) in the first period solves this maximization problem:

\[
\max_{B_1} \gamma \left[ \ln(y - g_0 - B_0 + q(B_1)B_1) + \beta \ln(y - g_1 - B_1) \right] + (1 - \gamma) \left[ \ln(y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0 - q(B_1) \left( \frac{\gamma}{1 - \gamma} \right) B_1 \right] + \beta \ln(y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) B_1)
\]

where \( q(B_1) = \beta \frac{y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right)(1 + \beta) B_1} \).

The first-order condition is:

\[
\gamma \left[ u'(c_{0L}^H) \left[ q'(B_1)B_1 + q(B_1) \right] - \beta u'(c_{1L}^H) \right] + (1 - \gamma) \left( \frac{\gamma}{1 - \gamma} \right) \left[ -u'(c_{0L}^H) \left[ q'(B_1)B_1 + q(B_1) \right] + \beta u'(c_{1L}^H) \right] = 0
\]

Using the Euler equation of the H types and simplifying:

\[
u'(c_{0L}^H) + \left( \frac{u'(c_{0L}^H)q(B_1) - \beta u'(c_{1L}^H)}{q'(B_1)B_1} \right) = u'(c_{0L}^H)
\]

This expression is important, because it defines a wedge between equating the two agents’ marginal utility of consumption that the planner finds optimal to maintain, given that the only instrument that it has to reallocate consumption across agents is the
debt. Notice that, since as noted earlier for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation implies that at the equilibrium price: \( u'(c^L_0)q(B_1) - \beta u'(c^L_1) > 0 \). Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports \( u'(c^L_0) > u'(c^H_0) \) or \( c^H_0 > c^L_0 \), and notice that from the budget constraints this also implies \( B_0 - q(B_1)B_1 > 0 \), which implies \( B_1/B_0 < 1/q(B_1) \). Furthermore, the latter implies that the optimal debt must be lower than any initial \( B_0 \) for any \( q(B_1) \geq 1 \), and also for "sufficiently high" \( q(B_1) \).

**Comparison with no-Debt Equilibrium**: Notice that since \( B_0 - q(B_1)B_1 > 0 \), the planner is allocating less utility to L type agents than those agents would attain without any debt. Without debt, and a tax policy \( \tau_t = g_t \), all agents consume \( y - g_t \) every period, but with debt L-types consume less each period given that \( B_1 > 0 \) and \( B_0 - q(B_1)B_1 > 0 \). Compared with these allocations, when the planner finds optimal to choose \( B_1 > 0 \) is because he is trading off the pain of imposing higher taxes in both periods, which hurts L types, against the benefit the H types get of having the ability to smooth using government bonds. Also, \( B_0 - q(B_1)B_1 > 0 \) highlights that there is a nontrivial role to the value of \( B_0 \), because if \( B_0 \) were zero \( B_1 \) would need to be negative which is not possible by construction. Hence, the model only has a sensible solution if there is already enough outstanding debt (and wealth owned by H type agents) that gives the government room to be able to improve the H type’s ability to smooth across the two periods, which they desire to do more the higher is \( B_0 \).

**Comparison with Sub-Optimal Debt Equilibrium**: By choosing positive debt, the government provides tax smoothing for L types. Given \( B_0 \) and the fact that \( B_0 - q(B_1)B_1 > 0 \), positive debt allows to lower date-0 taxes, which increases consumption of L types (since \( c^L_0 = y - g_0 - B_0 + q(B_1)B_1 \)). The same policy lowers the consumption of H types (since \( c^H_0 = y - g_0 + \left( \frac{1}{1 - \gamma(y)} \right) (B_0 - q(B_1)B_1) \)). Hence, debt serves to redistribute consumption across the two agents within the period. This also changes inter-temporal consumption allocations, with the debt reducing L types consumption in the second period and increasing H types consumption. Hence, with commitment to repay \( B_0 \), the debt will be chosen optimally to trade off these social costs and benefits of issuing debt to reallocate consumption atemporally across agents and intertemporally.  

It is also useful to notice that the demand elasticity of bonds is given by \( \eta = q(B_1)/(q'(B_1)B_1) \), so the marginal utility wedge can be expressed as \( \eta \left[ u'(c^L_0) - \frac{\beta u'(c^L_1)}{q(B_1)} \right] \) and the planner’s optimality condition reduces to:

\[
1 + \eta \left[ 1 - \frac{\beta u'(c^L_1)}{q(B_1)u'(c^L_0)} \right] = \frac{u'(c^H_0)}{u'(c^L_0)} \tag{A.17}
\]

Hence, the planner’s marginal utility wedge is the product of the demand elasticity of bonds and the L-type agents shadow value of being credit constrained (the difference \( 1 - \frac{\beta u'(c^L_1)}{q(B_1)u'(c^L_0)} > 0 \), which can be interpreted as an effective real interest rate faced by L-type agents that is higher than the return on bonds). The planner wants to use
positive debt to support an optimal wedge in marginal utilities only when the demand for bonds is elastic AND L-type agents are constrained.

A.3. Extension to Include Government Expenditure Shocks

Now consider the same model but government expenditures are stochastic. In particular, realizations of government purchases in the second period are given by the set \([g_1^1 < g_1^2 < ... < g_1^M]\) with transition probabilities denoted by \(\pi(g_1^i|g_0)\) for \(i = 1, ..., M\) with \(\sum_{i=1}^{M} \pi(g_1^i|g_0) = 1\).

A.3.1. Households. Preferences are now:

\[
\ln(c_0^i) + \beta \left( \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^i) \right) \quad \text{for } i = L, H \tag{A.18}
\]

Budget constraints are unchanged:

\[
c_0^L = y - \tau_0, \quad c_0^H = y - \tau_0 + b_0^H - q_b^H \tag{A.19}
\]
\[
c_1^L = y - \tau_1, \quad c_1^H = y - \tau_1 + b_1^H \tag{A.20}
\]

We still analyze cases where L types do not save, so they only consume what their budget constraints allow. For H types, the Euler equation becomes:

\[
q = \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \left( \frac{c_0^H}{c_1^H} \right) \tag{A.21}
\]

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, their Euler equation for bonds must satisfy:

\[
q > \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \left( \frac{c_0^L}{c_1^L} \right) \tag{A.22}
\]

A.3.2. Government. The government budget constraints are unchanged:

\[
\tau_0 = g_0 + B_0 - qB_1 \quad \tau_1 = g_1 + B_1
\]

The initial debt \(B_0 \geq 0\) is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

\[
y \left( \ln(c_0^L) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^L) \right) + (1 - y) \left( \ln(c_0^H) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^H) \right) \tag{A.23}
\]
A.3.3. Competitive equilibrium in the bond market. A competitive equilibrium in the bond market for a given supply of government debt \( B_1 \) is given by a price \( q \) that satisfies the market-clearing condition of the bond market: 

\[
b_1^H = B_1 / (1 - \gamma) .
\]

When \( b_1^L = 0 \) the equilibrium price can be derived from the H-types Euler equation.

We can solve the model in the same steps as before. First, rewrite the Euler equation of H types using their budget constraints, the government budget constraints and the market-clearing conditions:

\[
q = \beta \sum_{i=1}^{M} \pi(g_i^H | g_0) \left[ \frac{y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 - q \left( \frac{\gamma}{1-\gamma} \right) B_1}{y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1} \right] \tag{A.24}
\]

From here, we can solve again for the equilibrium price at a given supply of bonds:

\[
q(B_1) = \beta \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \right) \left( \sum_{i=1}^{M} \frac{\pi(g_i^H | g_0)}{y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1} \right) \tag{A.25}
\]

As \( \gamma \to 0 \) we converge again to the world where there is only an H type representative agent, but now the pricing formula reduces to the standard formula for the pricing of a non-state-contingent asset \( q(B_1) = \beta \left( \sum_{i=1}^{M} \pi(g_i^H | g_0) \frac{y - g_0}{y - g_1} \right) \). As \( \gamma \to 1 \) the equilibrium degenerates again into a situation where market clearing requires the demand of the infinitesimal small H type to rise to infinity.

The derivative of the price at any equilibrium with a positive bond price satisfies \( q'(B_1) < 0 \). To show this, define \( \Pi(B_1) = \sum_{i=1}^{M} \pi(g_i^H | g_0) \left( \frac{\gamma}{1-\gamma} \right) B_1 \) which yields \( \Pi'(B_1) = - \sum_{i=1}^{M} \frac{\pi(g_i^H | g_0)}{y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1} < 0 \). Then taking the derivative \( q'(B_1) \) and simplifying we get:

\[
q'(B_1) = \beta \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \right) \left[ \Pi'(B_1) - \beta \left( \frac{\gamma}{1-\gamma} \right) (\Pi(B_1))^2 \right] \tag{A.26}
\]

Since \( \Pi'(B_1) < 0 \) and positive \( c_0^H \) implies \( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 > 0 \), it follows that \( q'(B_1) < 0 \).

We can also gain some insight into the implicit risk premium (the ratio \( q(B_1) / \beta \)) and the related question of why the asset price can exceed 1 in this setup. Recall that in fact the latter was already possible without uncertainty when \( \gamma \) is large enough, because of the demand composition effect (higher \( \gamma \) implies by market clearing that the fewer H type agents need to demand more bonds per capita, so the bond price is increasing in \( \gamma \) and can exceed 1). The issue now is that, as numerical experiments
show, an increase in the variance of \( g_1 \) also results in higher bond prices, and higher than in the absence of uncertainty, and again for \( \gamma \) large enough we get both \( q(B_1) > 1 \) and \( q(B_1)/\beta > 1 \). The reason bond prices increase with the variability of government purchases is precautionary savings. Government bonds are the only vehicle of saving, and the incentive for this gets stronger the larger the variability of \( g_1 \). Hence, the price of bonds is higher in this stochastic model than in the analogous deterministic model because of precautionary demand for bonds, which adds to the effect of demand composition (i.e. the price is higher with uncertainty than without at a given \( \gamma \)).

A.3.4. Optimal Debt Choice. The government’s optimal choice of \( B_1 \) solves again a standard maximization problem:

\[
\max_{B_1} \left\{ \gamma \left[ \ln (y - g_0 - B_0 + q(B_1)B_1) + \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \ln (y - g_1 - B_1) \right] \right. \\
\left. + (1 - \gamma) \left[ \ln \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 - q(B_1) \left( \frac{\gamma}{1-\gamma} \right) B_1 \right) \right] \\
+ \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \ln (y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1) \right\}
\]

\[(A.27)\]

where \( q(B_1) \) is given by the expression solved for in the competitive equilibrium.

The first-order condition is:

\[
\gamma \left[ u'(c^L_0) \left[ q'(B_1)B_1 + q(B_1) \right] - \beta \sum_{i=1}^{M} \pi(g^i_1|g_0)u'(c^L_1) \right] \\
+ (1 - \gamma) \left( \frac{\gamma}{1-\gamma} \right) \left[ -u'(c^H_0) \left[ q'(B_1)B_1 + q(B_1) \right] + \beta \sum_{i=1}^{M} \pi(g^i_1|g_0)u'(c^H_1) \right] = 0
\]

\[(A.28)\]

Using the stochastic Euler equation of the H types and simplifying:

\[
u'(c^L_0) \left[ q'(B_1)B_1 + q(B_1) \right] - \beta \sum_{i=1}^{M} \pi(g^i_1|g_0)u'(c^L_1) = u'(c^H_0)q'(B_1)B_1 \quad (A.29)\]

\[
u'(c^L_0) + \frac{u'(c^L_0)q(B_1) - \beta \sum_{i=1}^{M} \pi(g^i_1|g_0)u'(c^L_1)}{q'(B_1)B_1} = u'(c^H_0) \quad (A.30)
\]

This last expression, compared with the similar expression of the planner without uncertainty, implies that in the planner’s view, the government expenditure shocks only matter to the extent that they affect the shadow price of the binding credit constraint of the L types. As before, since for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation would imply that at
the equilibrium price: \( u'(c_0^L)q(B_1) - \beta \sum_{i=1}^{M} \pi(g_1^i|g_0)u'(c_0^L) > 0 \). Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports \( u'(c_0^L) > u'(c_0^H) \) or \( c_0^H > c_0^L \), and notice that from the budget constraints this implies again \( B_0 - q(B_1)B_1 > 0 \), which implies \( B_1/B_0 < 1/q(B_1) \). Furthermore, the latter implies that the optimal debt must be lower than any initial \( B_0 \) for any \( q(B_1) \geq 1 \), and also for "sufficiently high" \( q(B_1) \). Thus the optimal debt choice again has an incentive to be lower than the initial debt.

A.4. Pricing Function: Stochastic Model with Default

We can also make an inference about what the pricing function looks like in the model with default risk, because with default we have a similar Euler equation, except that the summation that defines the term \( \Pi(B_1) \) above will exclude all the states of \( g_1 \) for which the government chooses to default on a given \( B_1 \) (and also at a given value of \( \gamma \)). That is, the term in question becomes \( \Pi^D(B_1) \equiv \sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1^i\left(\frac{\gamma}{1-\gamma}\right)B_1} \leq \Pi(B_1) \), and the pricing function with default risk is:

\[
q^D(B_1) = \frac{\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right) \Pi^D(B_1)}{1 + \left(\frac{\gamma}{1-\gamma}\right)\beta B_1 \Pi^D(B_1)} \leq q(B_1) \quad (A.31)
\]

Moreover, it follows from the previous analysis that this pricing function is also decreasing in \( B_1 \) \((q^D(B_1) < 0)\), and \( \Pi'(B_1) \equiv -\sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)}{(y-g_1^i\left(\frac{\gamma}{1-\gamma}\right)B_1)} \) is negative but such that \( \Pi'(B_1) \leq \Pi'(B_1) < 0 \). Also, it is clear from the above pricing functions that if the probability of default is small, so that there are only a few values of \( i \) for which \( d(B_1,g_1^i,\gamma) = 1 \) and/or the associated probability \( \pi(g_1^i|g_0) \) is very low, the default pricing function will be very similar to the no-default pricing function.

If we define the default risk spread as \( S(B_1,\gamma) \equiv [1/q^D(B_1,\gamma)] - [1/q(B_1,\gamma)] \), where \( \gamma \) has been added as an argument of the price functions because those prices also depend on variations in inequality, the spread reduces to the following expression:

\[
S(B_1,\gamma) = \left(\frac{1}{\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right)}\right) \left[\frac{1}{\Pi^D(B_1,\gamma)} - \frac{1}{\Pi(B_1,\gamma)}\right]
\]

Clearly since \( \Pi^D(B_1,\gamma) \leq \Pi(B_1,\gamma) \) the spread is non-negative, and it is strictly positive if there is default at equilibrium. The spread is increasing in \( B_1 \), because as the debt rises default is chosen optimally in more of the possible realizations of \( g_1^i \) and hence \( \Pi^D(B_1,\gamma) \) falls further below \( \Pi(B_1,\gamma) \), so that the gap between the reciprocals of these two terms widens. Note also that the spread is a multiple of the gap between these reciprocals, with the multiple given by \( 1/\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right) \).
As a result, the total date-0 resources available for consumption of the H-types \((y - g_0 + \left(\frac{\ln g_1}{\ln N}\right) B_0)\) have a first-order negative effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. The are also second order effects, because the equilibrium allocation of \(B_1\) also depends on that income measure, and thus \(\Pi^D(B_1, \gamma)\) and \(\Pi(B_1, \gamma)\) vary with it as well, but these are not considered here.

In terms of the effect of changes in \(\gamma\) on \(S\), notice that there are two effects. First, there is a negative effect because higher \(\gamma\) means that, for a given \(B_0\), the resources available for date-0 consumption of H types increase, since fewer H type agents need to demand enough initial bonds to clear the bond market, which means that per-capita each of the H types hold more date-0 bonds and have more bond income. Second, there is a positive effect because rising \(\gamma\) weakens default incentives as the welfare of the wealthy is valued more, and hence default is optimally chosen in more states, which increases \(\left[1/\Pi^D(B_1, \gamma) - 1/\Pi(B_1, \gamma)\right]\). Thus, in principle the response of the spread to increases in inequality is ambiguous. The weaker the response of the default probability to changes in \(\gamma\), however, the more likely it is that the first effect will dominate and the spreads will be a decreasing function of inequality.

A.5. Calibration of the Benchmark Model

The model is calibrated to annual frequency, and most of the parameter values are set so as to approximate some of the model’s predicted moments to those observed in the European data. The preference parameters are set to standard values: \(\beta = 0.96, \sigma = 1\). We also assume for simplicity that \(L\) types start with zero wealth, \(b^L_0 = 0\). We assume that

\[
\ln(g_1) \sim N \left( (1 - \rho_g) \ln(\mu_g) + \rho_g \ln(g_0), \frac{\sigma^2_\epsilon}{(1 - \rho^2_g)} \right).
\]

The parameters of this process are pinned down by estimating an AR(1) model with 1995-2012 data of the government expenditures-GDP ratio (in logs) for France, Germany, Greece, Ireland, Italy, Spain and Portugal. Given the parameter estimates for each country, we set \(\mu_g, \rho_g\) and \(\sigma_\epsilon\) to the corresponding cross-country average. This results in the following values \(\mu_g = 0.1812, \rho_g = 0.8802\) and \(\sigma_\epsilon = 0.017\). Given these moments, we set \(g_0 = \mu_g\) and use Tauchen (1986) quadrature method with 45 nodes in \(G_1 \equiv \{\bar{g}_1, \ldots, \bar{g}_1\}\) to generate the realizations and transition probabilities of the Markov process that drives expectations about \(g_1\).

2. \(\sigma = 1\) and \(b^L_0 = 0\) are also useful because, as we show in the Appendix, under these assumptions we can obtain closed-form solutions and establish some results analytically.
3. This specification allows us to control the correlation between \(g_0\) and \(g_1\) via \(\rho_g\), the mean of the shock via \(\mu_g\) and the variance of the unpredicted portion via \(\sigma^2_\epsilon\). Note that if \(g_0 = \mu_g\), \(\ln(g_1) \sim N(\ln(\mu_g), \frac{\sigma^2_\epsilon}{(1 - \rho^2_g)}).\)

4. As it is standard, we assume that \(g_1\) is an equally spaced grid with \(\log(\bar{g}_1)\) and \(\log(\bar{g}_1)\) located +/- 3 standard deviations from \(\log(\mu_g)\) respectively.
We abstain from setting a calibrated value for $\gamma$ and instead show results for $\gamma \in [0, 1]$. Note, however, that data for the United States and Europe suggest that the empirically relevant range for $\gamma$ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of $\gamma$ is useful we use $\gamma = 0.7$, which is the mid point of the plausible range.\(^5\)

Average income $y$ is calibrated such that the model’s aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households’ aggregate endowment must equal GDP net of fixed capital investment and net exports, since the latter two are not explicitly modeled. The average for the period 1970-2012 for the same set of countries used to estimate the $g_1$ process implies $y = 0.7883$. Note also that under this calibration of $y$ and the Markov process of $g_1$, the gap $y - g_1$ is always positive, even for $g_1 = \overline{g}_1$, which in turn guarantees $c^H_1 > 0$ in all repayment states.

We set the value of $B_0$ to match the median value of government debt for Eurozone countries reported in Table 1 of the paper. As we describe in the next section, even though the optimal $B_1$ (close to 15 percent of GDP for $\gamma \in [0.55, 0.85]$) is higher than those observed in the sovereign debt literature, in this two-period model the consumption smoothing mechanism induces a reduction in the optimal $B_1$ relative to the initial condition $B_0$ even in a deterministic version of the model with stationary government purchases. Under these assumptions, the optimal debt choice is decreasing in $\gamma$ and has an upper bound of $B_1 = B_0 / (1 + \beta)$ as $\gamma \rightarrow 0$.\(^6\) In addition, when default risk is introduced debt needs to be below the level that would lead the government to default in the second period with probability 1, and above the level at which either $c^L_0$ or $c^H_1$ become non-positive, otherwise there is no equilibrium.

The functional form of the default cost function is the following: $\varphi(g_1) = \varphi_0 + \frac{(\overline{g}_1 - g_1)}{y}$, where $\overline{g}_1$ is calibrated to represent an “unusually large” realization of $g_1$ set equal to the largest realization in the Markov process of government expenditures, which is in turn set equal to 3 standard deviations from the mean (in the process in logs).\(^7\) We calibrate $\varphi_0$ to match an estimate of the observed frequency of domestic default.

\(^5\) In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds but 50.4% have retirement accounts (which are very likely to own government bonds). These figures would suggest values of $\gamma$ ranging from 50% to 88%. In Europe, comparable statistics are not available for several countries, but recent studies show that the wealth distribution is highly concentrated and that the wealth Gini coefficient ranges between 0.55 and 0.85 depending on the country and the year of the study (see Davies et al. (2009)). In our model, if $b^L_0 = 0$, the Gini coefficient of wealth is equal to $\gamma$.

\(^6\) This occurs because as $y \rightarrow 0$, the model in deterministic form collapses to a representative agent economy inhabited by $H$ types where the optimal debt choice yields stationary consumption, $q_0 = 1/\beta$, and $B_1 = B_0 / (1 + \beta)$. In contrast, an infinite horizon, stationary economy yields $B_1 = B_0$ (see Appendix for details).

\(^7\) This cost function shares a key feature of the default cost functions widely used in the external default literature to align default incentives so as to support higher debt ratios and trigger default during recessions (see Arellano (2008) and Mendoza and Yue (2012)): The default cost is an increasing function of disposable income $(y - g_1)$. In addition, this formulation ensures that households’ consumption during a default never goes above a given threshold.
defaults. According to Reinhart and Rogoff (2011), historically, domestic defaults are about 1/4 as frequent as external defaults (68 domestic vs. 250 external in data since 1750). Since the probability of an external default has been estimated in the range of 3 to 5% (see for example Arellano (2008)), we estimate the probability of a domestic default at about 1%. The model is close to this default frequency on average when solved over the empirically relevant range of \( \gamma \)'s (\( \gamma \in [0.55, 0.85] \)) if we set \( \phi_0 = 0.004 \). Note, however, that this calibration of \( \phi_0 \) to target the default probability and the calibration of \( B_0 \) to the target described early needs to be done jointly by repeatedly solving the model until both targets are well approximated.

**A.6. Additional Figures: Benchmark Model**

Figure A.2 shows two panels with the optimal default decision for different values of \( g_1 \). The plots separate the regions where the government chooses to repay \( (d(B_1, g_1, \gamma) = 0 \) shown in white), where it chooses to default \( (d(B_1, g_1, \gamma) = 1 \) in green) and where the equilibrium does not exist (in blue).

![Figure A.2. Government default decision \( d(B_1, g_1, \gamma) \).](image)

The repayment region \( (d(B_1, g_1, \gamma) = 0) \) corresponds to the region with \( \bar{o}(B_1, g_1, \gamma) < 0 \). Hence, the government defaults at higher \( B_1 \) the lower a given \( \gamma \), or at higher \( \gamma \) the lower a given \( B_1 \). Moreover, the two plots show that when \( g_1 = \bar{g}_1 \) the government defaults for combinations of \( \gamma \) and \( B_1 \) for which it repays when \( g_1 = g_1 \). Thus, default occurs over a wider set of \( (B_1, \gamma) \) pairs at higher levels of government expenditures, and thus it is also more likely to occur.
We examine further the behavior of the default decision by computing the threshold value of $\gamma$ such that the government is indifferent between defaulting and repaying in period for a given $(B_1, g_1)$. These indifference thresholds ($\hat{\gamma} (B_1, g_1)$) are plotted in Figure A.3 against debt levels ranging from 0 to 0.4 for three values of government expenditures $\{\underline{g}_1, \mu g, \overline{g}_1\}$. For any given $(B_1, g_1)$, the government chooses to default if $\gamma \geq \hat{\gamma}$.

![Figure A.3. Default threshold $\hat{\gamma} (B_1, g_1)$](image)

Figure A.3 shows that the default threshold is decreasing in $B_1$. Hence, the government tolerates higher debt ratios without defaulting only if wealth concentration is sufficiently low. Also, default thresholds are decreasing in $g_1$, because the government has stronger incentives to default when government expenditures are higher (i.e. the threshold curves shift inward).\(^8\) This last feature of $\hat{\gamma}$ is very important to determine equilibria with debt subject to default risk. If, for a given value of $B_1$, $\gamma$ is higher than the curve representing $\hat{\gamma}$ for the lowest realization in the Markov process of $g_1$ (which is also the value of $g_1$), the government defaults for sure and, as explained earlier, there is no equilibrium. Alternatively, if for a given value of $B_1$, $\gamma$ is lower than the curve representing $\hat{\gamma}$ for the highest realization of $g_1$ (which is the value of $\overline{g}_1$), the government repays for sure and debt would be issued effectively without default risk. Thus, for the model to support equilibria with debt subject to default risk, the optimal debt chosen by the government in the first period for a given $\gamma$ must lie between these

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8. $\hat{\gamma}$ approaches zero for $B_1$ sufficiently large, but in Figure A.3 $B_1$ reaches 0.40 only for exposition purposes.
two extreme threshold curves. We show that this is the case later in this Section. Figure A.4 shows intensity plots of the equilibrium tax functions.

![Equilibrium Tax Functions](image)

Figure A.4. Equilibrium Tax Functions $\tau^d_1(B_1, g_1, \gamma)$.

Putting together these plots with those of the default decision in Figure A.2 illustrates the model’s distributional incentives to default from the perspective of tax policy. If the government defaults, the tax is $\tau_1 = g_1$, but if it repays the tax is $\tau_1 = g_1 + B_1$. Since all agents pay the same taxes but $L$ types do not collect bond repayments, the lower taxes under default provide a distributional incentive to default that is larger when wealth is more concentrated. Figure A.4 shows that, for given $g_1$, the repayment scenarios with higher taxes are more likely when a large fraction of households hold debt (low $\gamma$) and thus benefit from a repayment, or when the debt is low so that the distributional incentives to default are weak. Moreover, equilibria with higher taxes are more likely to be observed at low than at high levels of government expenditures, because default is far likely with the latter.

Figure A.5 plots the bond demand decision rules (for given $B_1$) of the $H$ types in the same layout as the bond prices (i.e. as functions of $\gamma$ for three values of $B_1$). This plot validates the intuition provided above about the properties of these agents’ bond demand function. In particular, as $\gamma$ increases, the demand for bonds of $H$ types grows at an increasing rate, reflecting the combined effects of higher per-capita demand by a smaller fraction of $H$-type agents and a rising default risk premium. Thus, the

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9. We do not include the bond decision rules for $L$ types because they are credit constrained (i.e. their Euler condition holds with inequality) and choose $b^L_t = b^L_0 = 0$. 

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convexity of these bond decision rules reflects the effects of wealth dispersion on demand composition and default risk explained earlier.

Figure A.6 shows the debt Laffer curves for five values of $\gamma$ in the [0.05,0.95] range. When $\gamma \geq 0.50$, $B_1^*(\gamma)$ is located at the maximum of the corresponding Laffer curve. In these cases, setting debt higher than at the maximum of the Laffer curve is suboptimal because default risk reduces bond prices sharply, moving the government to the downward sloping region of the Laffer curve, and setting it lower is also suboptimal because then default risk is low and extra borrowing generates more resources since bond prices change little, leaving the government in the upward sloping region of the Laffer curve. Thus, if the optimal debt has a nontrivial probability of default, the government’s debt choice exhausts its ability to raise resources by borrowing.

A.7. Sensitivity Analysis: Benchmark Model

This Section presents the results of a set of counterfactuals that shed more light on the workings of the model and also show some results for the case in which the social welfare function has biased weights that do not correspond to the fractions of $L$ and $H$ types in the economy. The sensitivity analysis studies how the main results are affected

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10. Each curve is truncated at values of $B_1$ in the horizontal axis that are either low enough for $c^L_1 \leq 0$ or high enough for default to be chosen for all realizations of $g_1$, because as noted before in these cases there is no equilibrium.
"Laffer" Curve $B_1 \ast q(B_1, \gamma)$

by changes in the initial debt $B_0$, initial government expenditures $g_0$, and the constant in the default cost function $\varphi_0$.

A.7.1. Lower Initial Debt Level $B_0$. Figure A.7 compares the optimal government debt and associated equilibrium bond prices, spreads and default probability under the original calibration with $B_0 = 0.35$ and a value that is 20% lower ($B_{0,L} = 0.28$).

Panel (i) shows that the optimal debt choice is always lower for the lower $B_0$, but the difference narrows as $\gamma$ rises. This occurs because at low $\gamma$ default risk plays a negligible role, and the demand composition effect implies lower per-capita demand for bonds from $H$ type agents and a smaller fraction of credit-constrained $L$ types, so the government wants to sell less debt. But once default risk becomes relevant, the optimal debt choice is about the same.

With the lower $B_0$, bond prices are uniformly lower albeit slightly (panel (ii)), spreads are increasing as $\gamma$ rises for a wider range of $\gamma$ and attain a lower maximum (panel (iii)), and the same is true for default probabilities (panel (iv)). Bond prices are lower with lower initial debt because for a given $\gamma$ this implies lower initial wealth of $H$ types ($b_{0,H}$), and in turn this requires a lower bond price to clear the market. This effect is stronger than two other effects that push bond prices in the opposite direction: First, the slightly lower debt $B_1$ that the government finds optimal to supply at lower levels of $B_0$. Second, the higher disposable income of households resulting from the
lower date-0 taxes needed to repay lower levels of $B_0$, which increases demand for bonds.

In terms of the implications for the empirically relevant range of $\gamma$ in the European data, these results show that at the lower $B_0$ the model continues to predict debt ratios of about 8 to 12%, but now at spreads that are only half (40 v. 100 basis points) and at default probabilities below 1 percent instead of 1 to 1.5 percent. Moreover, these results also show that spreads and default probabilities can display richer patterns than the ones found in the initial calibration.

A.7.2. Lower Initial Government Expenditures $g_0$. Figure A.8 compares the model’s equilibrium outcomes under the calibrated value of initial government expenditures, $g_0 = \mu_g = 0.181$, and a scenario in which $g_0$ is 1.5 standard deviations below the mean, $g_{0,L} = 0.171$. 

\[\text{Figure A.7. Changes in initial government sebt } B_0.\]
Like the reduction in $B_0$, the reduction in $g_0$ increases date-0 disposable income via lower date-0 taxes. They differ, however, in two key respects: First, changes in $g_0$ affect the expected level of government expenditures for $t = 1$, as reflected in changes in the transition probabilities which are conditional on $g_0$. Second, changes in $g_0$ do not affect the aggregate wealth of the economy and the initial bond holdings of $H$ types.

Panel (i) shows that the optimally debt is slightly higher with lower $g_0$, unless $\gamma$ is below 0.35. This reflects the fact that the lower $g_0$ allows the government to issue more debt in the initial period, because the likelihood of hitting states with sufficiently high $g_1$ for optimal default to occur in the second period is lower. This also explains why in panels (ii)-(iv), despite the higher optimal debt with the lower $g_0$, bond prices, default probabilities and spreads are lower. Moreover, optimal debt is about the same with lower $g_0$ as in the initial calibration for $\gamma \leq 0.35$ because at this low level of wealth concentration default risk is not an issue and the mechanism that we just described is irrelevant.

A.7.3. Fixed Cost of Default $\varphi_0$. Panels (i) – (iv) in Figure A.9 compare the equilibrium outcomes for the initial calibration of the default cost parameter ($\varphi_0 = 0.004$) with an scenario with a higher values ($\varphi_{0,H} = 0.02$).
Qualitatively, the changes in the equilibrium are in the direction that would be expected. Higher default costs increase the optimal debt and reduce bond prices, spreads and default probabilities. Quantitatively, however, the changes in optimal debt and bond prices are small, while the reduction in spreads and default probabilities are significantly larger.

**A.8. Additional Figures: Model with Biased Welfare Weights**

Adding default costs to the political bias setup ($\varphi(g_1) > 0$) makes it possible to support repayment equilibria even when $\omega \geq \gamma$. As Figure A.10 shows, with default costs there are threshold values of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega \geq \gamma$. 
It is also evident in Figure A.10 that the range of values of $\varepsilon$ for which repayment is chosen widens as $\gamma$ rises relative to $\omega$. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in $\varepsilon$, do not produce consumption dispersion larger than the value of $\hat{\varepsilon}$ associated with a given $(\omega, \gamma)$ pair. Intuitively, the consumption of $H$-type agents must not exceed that of $L$-type agents by more than what $\hat{\varepsilon}$ allows, because otherwise default is optimal.

Figure A.11 shows the default decision rule induced by the planner’s welfare gains of default, again as a function of $\omega$ and $\gamma$ for the same two values of $B_1$. The region in white corresponds to cases where $d(B_1, g_1, \gamma, \omega) = 0$, the green region corresponds to $d(B_1, g_1, \gamma, \omega) = 1$ and the blue region corresponds to cases in which there is no equilibrium.
This Figure shows that when the $\omega$ is low enough, the government chooses default, and for a given $\omega$ the default region is larger the lower is $\gamma$. Taxes and prices for given values of $B_1$ and $\omega$ are linked to the default decision and $\gamma$ as in the benchmark model and the intuition behind their behavior is straightforward.

### A.9. Additional Figures: Model with Foreign Investors

Figure A.12 shows the bond decision rules for domestic and foreign investors as the value of $\gamma$ and $B_1$ change.
Figure A.12. Decision Rule Domestic and Foreign Agents

Figure A.13 presents the optimal domestic and foreign demand (Panel (i)) and the optimal debt issuance of the government (Panel (ii)). This Panel also marks with red squares the debt levels where there is positive risk of default and shows that the feature of equilibrium domestic default is also robust to the introduction of foreign lenders.
Panel (i) of Figure A.13 shows that domestic (foreign) demand is an increasing (decreasing) function of $\gamma$. Panel (ii) of Figure A.13 shows that the debt choice of the government is a concave function of $\gamma$. This is a direct result of default incentives. When $\gamma$ is low the government has strong incentives to default of foreign lenders, so debt issuance remains low. As $\gamma$ increases and domestic households increase their demand for sovereign debt, forces for a “foreign” default decrease and higher debt levels are attainable. As higher levels of $\gamma$, “domestic” default incentives constraint the government to increase the debt further and the optimal choice decreases.

A.10. Details for the Model with Two Assets

We now extend the baseline model by allowing domestic agents to save using a risk-free asset, in addition to government bonds. In particular, agents have access to a non-stochastic production technology $y^i_t = z(k^i_t)^{\theta}$ with $0 < \theta < 1$, where $y^i_t$ is total output and $k^i_t$ is capital for agent of type $i$ in period $t$, respectively. The initial aggregate level of capital is denoted by $K_0$. As before, there is a fraction $\gamma$ of $L-\text{type}$ agents that are endowed with $b^L_0$ and $k^L_0$ units of domestic sovereign debt and capital respectively. A fraction $(1-\gamma)$ of agents are of the $H-\text{type}$ and have endowments given by $b^H_0 = \frac{b^H_0 - \gamma b^L_0}{1-\gamma}$ and $k^H_0 = \frac{K_0 - \gamma k^L_0}{1-\gamma}$. We assume that capital depreciates at rate $\delta$. At
period 0, agents choose how much of their savings they want to allocate to public bonds $b_1^i \geq 0$ and capital $k_1^i \geq 0$.

The assumptions we made above to introduce the second asset, particularly the curvature of the production function, serve the purpose of supporting a well-defined portfolio choice for private agents (i.e. the fraction of wealth to allocate to bonds and capital), which is in turn useful to solve the government’s problem.\footnote{This statement is of course conditional on parameters supporting the interesting case in which at least some agents do hold both assets at equilibrium.} This formulation is also helpful because it approximates the cases in which agents could buy a foreign risk-free asset or a domestic asset that is in zero net supply (and is also risk free). If, for example, agents could buy a foreign risk-free asset, we could obtain well defined portfolios by introducing adjustment costs.

The agents’ budget constraint in the initial period is:

$$c_0^i + q_0 b_1^i + k_1^i = z(k_0^i)\theta^i + k_0^i (1 - \delta) + b_0^i - \tau_0 \text{ for } i = L, H.$$  \hfill (A.32)

The budget constraints in period 1 for the case of no-default and default are:

$$c_1^i,d=0 = z(k_1^i)\theta^i + k_1^i (1 - \delta) + b_1^i - \tau_1 \text{ for } i = L, H.$$ \hfill (A.33)

$$c_1^i,d=1 = (1 - \varphi(g_1))z(k_1^i)\theta^i + k_1^i (1 - \delta) - \tau_1 \text{ for } i = L, H.$$ \hfill (A.34)

The first-order condition with respect to $b_1^i$ is

$$u'(c_0^i) \geq (\beta/q_0)Eg_1\left[u'(c_1^i,d=0)(1 - d_1)\right], \text{ if } b_1^i > 0.$$  

As in the benchmark, the marginal benefit of an extra unit of domestic bond is positive only in those states where the government chooses to repay. Similarly, the first-order condition with respect to capital is given by

$$u'(c_0^i) \geq \beta Eg_1\left[u'(c_1^i,d=0)(1 - d_1) + u'(c_1^i,d=1)d_1\right]\left[\theta z(k_1^i)\theta^{-1} + 1 - \delta\right], \text{ if } k_1^i > 0.$$  

Since the asset is risk-free, the marginal benefit of an extra unit of capital is positive in all possible future states of the world. However, it will be more valuable in those states where the government chooses to default since the marginal utility is higher. These first-order conditions also show that the level of government debt $B_1$ (as well as the initial wealth dispersion $\gamma$) influence the individual asset demand through the default decision directly, and indirectly through the bond price.

When the borrowing constraint on bonds binds, this introduces a wedge between the agents’ expected marginal product of capital and the expected return on the bond. However, if both decisions are interior (which is likely to happen for $H$—type agents), agents equalize the expected return across assets, which implies:

$$\frac{q_0}{[\theta z(k_1^i)\theta^{-1} + 1 - \delta]^{-1}} = \frac{Eg_1\left[u'(c_1^i,d=0)(1 - d_1)\right]}{Eg_1\left[u'(c_1^i,d=0)(1 - d_1) + u'(c_1^i,d=1)d_1\right]}.$$  \hfill (A.35)
This equation shows that when capital and bond decisions are interior, the wedge between the price of the bond and the marginal productivity of capital is driven uniquely by default risk (weighted by marginal utility in each state of the world). In the particular case in which in addition we are in a combination of $B_1$ and $\gamma$ where there is no default risk, the optimal individual choice of capital is given by:

$$\frac{1}{q_0} = (\alpha z(k^i_1)^{\alpha-1} + 1 - \delta)$$

$$\Rightarrow k^i_1 = \left[ \frac{\alpha z q_0}{1 - q_0(1 - \delta)} \right]^{1/\alpha}.$$

The demand for capital increases with the price of the bond, or falls with the interest rate, as in standard investment models.

In this environment, the government faces similar budget constraints to those presented in the benchmark model. However, when choosing the optimal level of debt and whether to default or not, it takes into account not only the distribution of bond holdings across agents but also the distribution of capital. In particular, at $t = 1$, the repayment value is given by:

$$W_{d=0} = \gamma u(z(k^L_1)^{\theta} + (1 - \delta)k^L_1 - g_1 + b^L_1 - B_1) + (1 - \gamma)u(z(k^H_1)^{\theta} + (1 - \delta)k^H_1 - g_1 + b^H_1 - B_1)$$

and the value of default is

$$W_{d=1} = \gamma u(z(k^L_1)^{\theta}(1 - \varphi(g_1)) + (1 - \delta)k^L_1 - g_1) + (1 - \gamma)u(z(k^H_1)^{\theta}(1 - \varphi(g_1)) + (1 - \delta)k^H_1 - g_1).$$

Note that in each case, the individual capital and bond decisions $\{b^i_1, k^i_1\}$ are a function of $B_1$ and $\gamma$. This implies that the variance of consumption, a key moment to determine the differential value of repayment v. default, is a function of the amount of debt the government has issued in period 0.

The model seems simple, but its solution needs to be obtained numerically. To make this case as close as possible to the benchmark model, we assume that $k^i_0 = K_0$ (i.e. all initial heterogeneity is in initial bond holdings). Still, this results in heterogeneous bond and capital holdings in the second period. We set $z$ to normalize GDP to 1, $\theta$ equal to 0.33 (a standard value), $\delta = 0.10$ and set $K_0$ so the capital to output ratio is equal to 2. All other parameters are set equal to those in the benchmark.

Figure A.14 presents the decision rules of the model. The top panels correspond to those for the $H$—type and the bottom panels to those for the $L$—type. They are truncated for combinations of $B_1$ and $\gamma$ where the equilibrium does not exist because the government defaults for all possible realizations of $g_1$. 

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As in the benchmark model, $L$--type agents are at the borrowing constraint for all values of $B_1$. Their capital choice is determined completely by the first-order condition for capital. As we get close to a region with default risk, their demand for capital decreases since the marginal benefit of an extra unit of savings decreases, because they expect to receive a transfer in the case of a default. On the other hand, $H$--type agents are never at the borrowing constraint, so their choice of capital is given by equation (A.35). As in the benchmark economy with one asset, while default risk is low ($B_1 = B_{1,L}$), the bond price increases with $\gamma$ to induce the $H$--type to demand a decreasing fraction of their initial bond position (but still increasing in $\gamma$). This increase in bond prices drives the increasing demand for capital for the $H$--type when $B_1 = B_{1,L}$. As we move to higher levels of $B_1$, similar factors are at play but since default risk is not trivially small, bond prices do not rise as fast, inducing a relatively flat demand for capital with slightly increasing demand for domestic bonds for $H$--types with a decreasing demand for capital for $L$--types.
Figure A.15. Equilibrium model with two assets. $mpk^H$ denotes the marginal product of capital for the $H$—type and is given by $\theta z(k^H)^{\theta-1} + 1 - \delta$. The spread is computed as $1/q(B^1\gamma) - mpk^H$.

Figure A.15 presents the equilibrium functions as they vary with $\gamma$. As before, we present the optimal debt choice of the government, and the associated bond prices, спreads (measured against the marginal product of capital of the $H$—type) and the default probability.12 This figure shows that the results of the benchmark model are robust to the introduction of an additional asset.

Panel (i) shows that optimal debt falls as $\gamma$ increases and Panel (iv) that default risk is not zero for $\gamma > 0.20$ (non negligible for $\gamma > 0.50$). Importantly, the probability of a default event is increasing in the initial level of wealth inequality. As default risk increases, the wedge between the marginal product of capital and the equilibrium bond price increases driving up the spreads (see Panels (ii) and (iii)). Different from our baseline model, bond prices remain below 1, implying positive real interest payments for all values of $\gamma$.

Figure A.16 compares the equilibrium functions of the benchmark model with those of the economy with two assets.

12. We measure spreads against the marginal product of capital of the $H$—type since the wedge is only driven by default risk. In the case of the $L$—type agents there is an additional component that arises from the multiplier on the non negativity constraint on domestic bonds.
The model with two assets sustains higher levels of debt, but as default risk increases the government chooses to reduce debt issuance faster in the economy with capital than in the benchmark. Even though we observe a lower default probability and spreads, the previous result derives from the fact that lower levels of debt are sustainable when agents have access to another asset. The cost of a default is not as high for the $H$—types and a default reduces consumption dispersion that increases faster with $\gamma$ in the economy with capital than in the benchmark since initial dispersion is amplified via capital.

References


