OPTIMAL MONETARY POLICY AND FINANCIAL STABILITY IN A NON-RICARDIAN ECONOMY

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Abstract
I present a model with Discontinuous Asset Market Participation (DAMP), where all agents are non-Ricardian, and where heterogeneity among market participants implies financial-wealth effects on aggregate consumption. The implied welfare criterion shows that financial stability arises as an additional and independent target, besides inflation and output stability. Evaluation of optimal policy under discretion and commitment reveals that price stability may no longer be optimal, even absent inefficient supply shocks: some fluctuations in output and inflation may be optimal as long as they reduce financial instability. Ignoring the heterogeneity among market participants may lead monetary policy to induce substantially higher welfare losses. (JEL: E12, E44, E52)

Keywords: DAMP, Optimal Monetary Policy, Perpetual Youth, Financial Stability, DSGE Model, Asset Prices, LAMP.

1. Introduction

The burst of the “dotcom” bubble in 2001 and the recent global financial meltdown highlighted that developments in asset markets can be of great relevance for the business cycle, and revived a theoretical debate about the desirability that central banks be directly concerned with financial stability. To evaluate this concern from a welfare perspective, I develop a tractable dynamic stochastic general equilibrium model of a non-Ricardian economy populated by heterogeneous agents that stochastically cycle between zones of activity and inactivity in asset markets.
In particular, the economy is populated by ex-ante identical infinitely-lived agents that face discontinuous asset market participation (DAMP): at the beginning of each period, each agent finds out her current status in asset markets. If inactive, she behaves as a “rule-of-thumb”, consuming only her disposable labor income. If active, she trades in asset markets to smooth consumption, but subject to a finite planning horizon, as in each period she faces a constant probability of switching to the inactive type. This economy is therefore non-Ricardian for two reasons: “rule-of-thumbers” are non-Ricardian because they do not smooth consumption in asset markets, “market participants” because they face a finite planning horizon, as in a “perpetual-youth” model.

The model therefore generalizes the limited asset market participation (LAMP) framework by introducing an additional layer of agent heterogeneity in the economy – besides “active” versus “inactive” – related to the cross-sectional distribution of financial wealth among market participants. This specific type of heterogeneity gives rise to financial-wealth effects on aggregate consumption of the same kind as those arising in perpetual-youth models, and makes asset-price dynamics non-neutral for the equilibrium allocation. The resulting DAMP framework therefore nests as polar cases two other popular models extensively used to study monetary business cycles: i) the standard “representative-agent” model, where infinitely-lived identical agents are active in asset markets at all times, and ii) the discrete-time “perpetual-youth” (PY) model, where all agents are active at all times, but they are finitely lived and thus heterogeneous. The latter model, in particular, provides a natural theoretical characterization of the behavior of active agents in a DAMP economy: they face a positive probability of being replaced by previously inactive agents, they discount future utility to account for both impatience and transition probabilities, they have an incentive to enter an insurance contract à la Blanchard (1985) in order to smooth consumption while market participants.

Analyzing welfare-maximizing policy and the implied trade-offs in the DAMP model, thus, requires to deal with a social welfare function that aggregates the utility flows of an infinite set of heterogenous agents, as in PY models. One contribution of this paper is to show how to treat the heterogeneity among market participants to characterize a second-order approximation of social welfare in terms of aggregate variables only: the cross-sectional consumption dispersion among active agents is shown to be proportional to squared aggregate financial wealth. The resulting criterion requires financial stability to be an additional and independent target of a welfare-maximizing policy maker, besides inflation and output stability. The relative weights on the three targets reflect the two layers of heterogeneity of this economy: between agent

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1. For monetary policy analyses within the LAMP framework see, among others, Galí et al (2007) and Bilbiie (2008).
3. This complication is one reason why, to my knowledge, no normative analysis in a linear-quadratic framework is available within perpetual-youth models.
types and within the set of market participants. As a consequence, an endogenous trade-off arises between output and price stability on the one hand, and financial stability on the other: price stability may no longer be an optimal monetary policy regime. Even conditionally only on productivity shocks, some fluctuations in inflation and the output gap may be optimal as long as they increase financial stability.

From a methodological perspective, the model developed in this paper builds on two strands of the literature. The first one was initiated by the seminal papers of Yaari (1965) and Blanchard (1985), which introduced and characterized the PY model in a continuous-time framework. Since then, discrete-time baseline versions have been widely used in the literature to study a variety of topics, both in closed and open-economy models or as building blocks of more elaborate frameworks, to account for demographic factors. In contrast to these studies, all of which use the perpetual-youth structure to characterize the implications of finite lifetimes for economic policy, this paper shows that it captures quite naturally the transient nature of the “active” agent type – while the agent’s lifetime itself is infinite – and describes its optimal behavior.

The second strand, initiated by the empirical observation of Campbell and Mankiw (1989) and the theoretical suggestions of Mankiw (2000), features an increasing number of New-Keynesian models formally characterizing limited asset market participation as the interplay between two different types of agents in the economy: optimizing (Ricardian) agents smoothing consumption over an infinite horizon through asset-market trading, and hand-to-mouth (non-Ricardian) agents consuming instead their labor income period by period. As also discussed by Bilbiie (2008), these LAMP models essentially build into the dynamic New-Keynesian framework the exogenous segmentation in asset markets typical of the literature analyzing the “liquidity effects” of monetary policy, such as Alvarez, Lucas and Weber (2001). Accordingly, in these models such partition of the economy is “static”: an agent of a given type stays of that type forever. This paper, in contrast, allows for a stochastic transition between the two agent types analogous to that implied by the endogenously-segmented market structure discussed in Alvarez, Atkeson and Kehoe (2002).

Indeed, I interpret the transition between agent types – that in the DAMP model is taken as exogenous – as a reduced-form representation of the interaction among (i) a costly decision to participate in asset markets, (ii) some liquidity constraint and (iii) idiosyncratic uncertainty about individual income. This interaction – microfounded by Alvarez, Atkeson and Kehoe (2002) – implies that some agents optimally decide to pay the participation cost and engage in asset-market trading to smooth consumption intertemporally, while the remaining ones decide to be inactive and simply consume...
their own endowment (previously sold for cash). In every period, idiosyncratic shocks to individual endowments make it optimal, for some agents, to revise their decision and switch between the zones of activity and inactivity, thereby implying stochastic transition between states.

An alternative interpretation might focus on the feature that infinitely-lived agents take intertemporal consumption decisions over a finite planning horizon. A microfounded rationale for such implication is provided by McKay et al. (2015), within a heterogenous-agent model addressing the forward-guidance puzzle. In one specification of their model economy, uninsurable idiosyncratic income risk makes agents randomly switch between employment and unemployment, with the latter further implying a binding liquidity constraint. In this environment, employed agents discount the future more than implied by their time-discount rate, as they also take into account the probability of suddenly becoming unemployed, whereby they would be unable to borrow and forced to consume only their home production. This effectively shortens the agents’ planning horizon, despite their infinite lifetime, thus implying a “discounted euler equation” analogous to that arising in PY models. Under this interpretation, the transition probabilities in the DAMP model would capture the transition in and out of unemployment, where the latter also implies inactivity in asset markets.

A third possible interpretation might regard the DAMP model as an approximate description of consumers’ behaviour over the life cycle, in an economy where finitely-lived agents face (i) borrowing constraints, (ii) hump-shaped expected income profiles and (iii) uninsurable idiosyncratic labor-income risk, as in Gourinchas and Parker (2002). Under this interpretation, the inactive agents are those in the early stage of their life cycle – not smoothing consumption over time because of self-imposed borrowing constraints and only barely saving for precautionary motives – while the active agents are those in the late stage of their life cycle – whose behaviour is instead consistent with the permanent-income hypothesis. Accordingly, under this interpretation, the transition probabilities in the DAMP model would capture the expected duration of the two stages of the life cycle.

As the three interpretations above suggest, therefore, the DAMP model is in principle suitable to address different kinds of issues (with respect to the one analyzed in this paper), in economies where idiosyncratic uncertainty implies occasionally binding liquidity constraints for some agents.

More broadly, this paper is also related to a large literature spurred by the stock-market burst of 2001, evaluating to what extent a central bank is (and should be)
concerned with stock-price dynamics. This literature analyzes the implications for monetary policy of fluctuations in asset markets that affect real activity through both
a supply-side channel – induced by the financial accelerator, as in Bernanke and
Gertler (1999) and Cecchetti et Al. (2000) – as well as a demand-side one – through
financial-wealth effects induced by heterogeneity among finitely-lived agents, as in Di
participation to financial markets of this paper introduces an analogous demand-side
channel in a more general class of models, where agents are heterogenous both between
active and inactive types and within the active type. The resulting framework is used to
complement the positive analysis in Nisticò (2012) and Airaudo et al (2015), and the
empirical analysis in Castelnuovo and Nisticò (2010), with a complete welfare analysis
of the links between monetary policy and financial stability.

The paper is structured as follows. Section 2 characterizes the DAMP economy,
and derives the welfare-based loss function as a second-order approximation of social
welfare. Section 3 analyzes the stabilization trade-offs and the optimal monetary policy
design. Section 4 concludes.

2. The DAMP Economy

Consider an economy populated by a continuum of infinitely-lived agents in the
interval \([0, 1]\). A mass \((1 - \vartheta)\) of these agents is financially inactive, consuming her
labor income period by period; I also refer to these agents as “rule-of-thumbers”. The
remaining mass \(\vartheta\) of agents trades in asset markets to save/borrow and smooth
consumption; I refer to these agents as “market participants”, or “active”. The LAMP
literature (e.g. Galí et al., 2007 and Bilbiie, 2008) typically assumes that a given agent
cannot change type: once inactive, always inactive. I generalize this framework by
allowing for stochastic transition between these two types of agent. Each agent’s type
evolves over time as an independent two-state Markov chain: each period, each agent
learns whether or not she is going to participate in asset markets, where the relevant
probability is only dependent on the agent’s current state. In particular, with probability
\(\xi \in [0, 1]\), a market participant becomes inactive, while with probability \(1 - \xi\) she
remains active. Analogously, with probability \(\varrho \in [0, 1]\), a “rule-of-thumber” becomes
active, while with probability \(1 - \varrho\) she remains inactive.

Define a “cohort” as the set of agents experiencing a transition between types in the
same period: given the continuum of agents populating the economy, the size of each
individual agent is negligible with respect to her cohort. By the law of large numbers,
in each period a mass \(\vartheta \xi\) of “market participants” becomes inactive, and a mass \((1 - \vartheta)\varrho\)
of “rule-of-thumbers” becomes active. Accordingly, the size of each cohort declines
deterministically: at time \(t\), the mass of agents that became active at time \(j\) and have
not yet switched type is \(m_{p,t}(j) \equiv \vartheta \xi (1 - \xi)^{t-j}\), while the mass of agents that became

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inactive at time $k$ and have not yet switched type is $m_{r,t}(j) \equiv (1 - \vartheta)\varrho(1 - \varrho)^{t-k}$, where subscript $p$ denotes market participants and $r$ rule-of-thumbers.

I focus on an economy where the population shares of the two agent types remain constant over time, in which therefore $\varrho(1 - \vartheta) = \vartheta \xi$: at each time, the mass of active agents leaving financial markets is equal to the mass of rule-of-thumbers joining them. It follows that, in general, $\vartheta$ and $\xi$ must satisfy the following restriction:

$$\vartheta \xi \leq 1 - \vartheta, \tag{1}$$

requiring the inactive set to be at least large enough to provide a replacement for all active agents switching type, so that all agents are infinitely lived and population is always equal to $1$.\footnote{Violating restriction (1) implies that either i) population size is constant but agents are not infinitely lived, as a mass $\vartheta(1 + \xi) - 1$ of agents actually dies each period, or ii) all agents are infinitely lived but population is not constant, as a mass $\vartheta(1 + \xi) - 1$ of new agents is born, thereby increasing population size to $\vartheta(1 + \xi) > 1$ and reducing the relative size of the set of active agents to $\vartheta = 1/(1 + \xi)$, which satisfies (1) with equality. In both cases, $\varrho = 1$.}

Since they face a positive probability of becoming inactive, each cohort of “market participants” has a different stock of accumulated wealth, depending on its longevity in the type. The model therefore introduces an additional layer of agents heterogeneity: agents are not only heterogenous between the sets of active and inactive – as in the LAMP framework – but they also are within the set of active – as in a PY model. This additional layer of heterogeneity implies, in equilibrium, a financial-wealth effect on aggregate consumption of the same kind as that arising in a PY model. Characterizing the set of active agents, therefore, requires to keep track of the cross-sectional distribution of financial wealth. The rule-of-thumbers, instead, are all identical, as they all consume their entire labor income, regardless of their longevity as inactive. Accordingly, the economy-wide aggregate level of generic variable $X$ is computed as the mass-weighted average

$$X_t \equiv \vartheta X_{p,t} + (1 - \vartheta)X_{r,t} = \vartheta \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} X_{p,t}(j) + (1 - \vartheta)X_{r,t}, \tag{2}$$

where $X_{p,t} \equiv \sum_{j=-\infty}^{t}(m_{p,t}(j)/\vartheta)X_{p,t}(j) = \sum_{j=-\infty}^{t}\xi(1 - \xi)^{t-j} X_{p,t}(j)$ is the average value of $X$ across all “market participants”, $j \in (-\infty, t]$ indexes the discrete set of cohorts of the latter at time $t$, and $X_{r,t}$ is the average level across all “rule-of-thumbers” (which is the same for each of them).

Notice that, although lifetime is infinite for each agent, each “market participant” takes intertemporal consumption decisions subject to a planning horizon which is instead finite, thereby considering that she will still be active after $n$ periods with probability $(1 - \xi)^n$.\footnote{Indeed, a market participant anticipates that, upon becoming inactive – and regardless of the consumption decisions taken while still active – she will behave as a rule-of-thumber and consume her entire labor income in each period unless she switches back to the active-type. Therefore, the relevant time horizon for her consumption decisions is $n$. This is consistent with the infinite lifetime assumption, as the finite horizon is simply a planning horizon for the consumption of an infinite-lived agent.} Thus, the problem of a market participant is isomorphic to that of a market participant in an infinite-lived economy with a finite horizon of planning.

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of an agent in a PY model à la Blanchard (1985). In particular, market participants also have an incentive to enter an insurance contract which grants them an extra-return on financial assets in exchange for the accumulated wealth at the time in which they are drawn to be replaced. Without such insurance contract active agents would need to consume (pay back) the accumulated wealth (debt) entirely in the period in which they switch type; entering the contract, instead, allows them to insure against the idiosyncratic risk of replacement, and consume (pay back) their wealth (debt) smoothly over time while still active. Therefore, all agents in this economy are non-Ricardian: rule-of-thumbers because they do not trade in asset markets, market participants because they face a finite planning horizon.

The resulting DAMP framework nests as polar cases most models extensively used in the monetary policy New-Keynesian literature. If $\theta \in (0, 1)$ and $\xi = 0$, the model becomes the standard LAMP specification of Gali et al. (2007) and Bilbiie (2008), among others: all agents are infinitely lived and a share of them are inactive in asset markets, but those agents will never become active. If $\theta = 1$ and $\xi = 0$ the model instead nests the standard “representative-agent” framework, where all agents are both infinitely lived and continuously active in asset markets. Finally, if $\theta = 1$ and $\xi \in (0, 1)$ – thus violating – the model collapses to a standard “perpetual-youth” framework, where the transition probability $\xi$ reflects expected lifetime: all agents are active, but they still face a positive probability of exiting the market (dying) and being replaced by newcomers (newborn).

### 2.1. The Decentralized Allocation

All agents in the economy have Cobb-Douglas preferences over consumption $C$ and leisure $1 - N$:

$$U_{s,t}(h) \equiv \log C_{s,t}(h) + \delta \log (1 - N_{s,t}(h)),$$

horizon when taking intertemporal consumption decisions as a market participant is the expected duration of the “active regime”: $1/\bar{\xi}$. Analogously, the expected duration of the “inactive regime” is $1/\theta$, but the effective planning horizon for consumption decisions is 1 period, since a rule-of-thumber cannot transfer resources over time through financial markets.

13. An alternative assumption would be to allow inactive agents to passively roll over their assets until they switch back into the active type. This would complicate the analysis considerably (as it would require to keep track of the distribution of wealth within both sets of agents) without affecting the qualitative implication that the transition between agent types induces wealth-effects of financial instability on aggregate spending. See also Section 2.1.

14. Ascari and Rankin (2007) argue that PY models with endogenous labor supply should feature GHH preferences, to avoid issues related to the negative labor supply of oldest generations. I acknowledge this result – which might affect active agents in the DAMP model as well – but choose to use standard preferences, for the sake of comparability.
Regardless of their specific longevity as rule-of-thumbers, all inactive agents face the same static problem: they seek to maximize their period utility \( U_{r,t} \) subject to the following budget constraint:

\[ C_{r,t} = W_t N_{r,t}, \]

where \( W_t \) denotes the real wage. Solution to this problem yields a labor supply schedule

\[ N_{r,t} = 1 - \delta \frac{C_{r,t}}{W_t}, \quad (4) \]

implying that, at equilibrium, inactive agents work a constant amount of hours: \( N_{r,t} = (1 + \delta)^{-1} \).

In addition to supplying labor services and demanding consumption goods, “market participants” also demand two types of financial assets: nominal state-contingent bonds \( B_{t+1}^* \equiv P_{t+1} B_{t+1} \) and equity shares \( Z_{t+1}(i) \) issued by a continuum of monopolistic firms, indexed by \( i \in [0, 1] \) and selling at real price \( Q_t(i) \).

At time \( t_0 \), agents that have been active since period \( j \leq t_0 \) seek to maximize

\[ E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} (1 - \xi)^t U_{p,t}(j) \right\}, \]

where future utility flows are discounted to account for both impatience (\( \beta \)) and the probability of becoming inactive in the next period (\( \xi \)). The maximization problem is subject to a sequence of budget constraints (expressed in real terms) of the form:

\[ C_{p,t}(j) + E_t \left\{ F_{t,t+1} \frac{P_{t+1}}{P_t} B_{t+1}(j) \right\} + \int_0^1 Q_t(i) Z_{t+1}(j,i) \, di \]

\[ \leq \frac{1}{1 - \xi} \Omega_t(j) + W_t N_{p,t}(j) - T_t(j), \quad (5) \]

where \( W_t N_{p,t}(j) - T_t(j) \) is real disposable labor income, \( F_{t,t+1} \) is the stochastic discount factor for state-contingent assets, and the real financial wealth carried over from the previous period is

\[ \Omega_t(j) = \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_{t}(j,i) \, di + B_t(j), \quad (6) \]

including the pay-off on the equity portfolio and on the contingent claims. As discussed earlier, in order to smooth consumption over the duration of the active regime, “market participants” enter an insurance contract à la Blanchard (1985), which grants them an extra-return \( \frac{\xi}{1 - \xi} \) on financial assets in exchange for the accumulated wealth at the time in which they are drawn to be replaced.

It is useful, at this point, to further characterize the partition of agents interacting in asset markets in each period \( t \): call “old traders” those that are active since at least one period (i.e. \( j \leq t - 1 \)), and “newcomers” those that became active in the current period (i.e. \( j = t \)). The latter were previously inactive, and I therefore assume – in analogy with Blanchard (1985) – that they enter with zero holdings

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15. This Section describes only the main features of the model. For details, please refer to the Online Appendix. To save on notation, subscripts \( p \) and \( r \) are used only for consumption and hours worked, that enter the problems of both types.
The different longevity among “market participants” implies a non-degenerate distribution of financial wealth across cohorts, and a related cross-sectional distribution of individual consumption. I assume that the public authority is able to directly affect such distribution only in the steady state, by means of an appropriate redistribution scheme:

\[ T_t(j) = \tilde{T}_t + \gamma(j) \mathcal{R}_t. \]

The first component is common across cohorts, while the second one is a generation-specific transfer, where \( \mathcal{R}_t \) is an appropriate scaling factor and \( \gamma(j) \) defines whether the transfer is a fee – \( \gamma(j) > 0 \) – or a subsidy – \( \gamma(j) < 0 \). \(^{17}\)

The nominal gross return \((1 + r_t)\) on a safe one-period bond paying off one unit of currency in period \( t + 1 \) with probability 1 is defined by:

\[ (1 + r_t)E_t \{ \mathcal{F}_{t,t+1} \} = 1. \] (7)

Using the individual equilibrium conditions to solve equation (5) forward, and aggregating across all market participants, one can derive the equilibrium dynamics of their average consumption:

\[ C_{p,t} = \frac{\xi \sigma}{\beta(1 - \xi)} E_t \{ \mathcal{F}_{t,t+1}(1 + \pi_{t+1})(\Omega_{t+1} - \gamma^{nc}) \} + \frac{1}{\beta} E_t \{ \mathcal{F}_{t,t+1}(1 + \pi_{t+1})C_{p,t+1} \}. \] (8)

where \( 1 + \pi_t \equiv P_t/P_{t-1} \), \( \gamma^{nc} \equiv -\gamma(t)(1 - \xi)^{-1} > 0 \) is the transfer received by newcomers upon becoming active, and \( \sigma \equiv [1 - \beta(1 - \xi)]/(1 + \delta) \). The first term in the equation above captures the financial-wealth effect, and can be traced back to the cross-sectional consumption dispersion, as measured by the wedge between the average consumption of old traders \( (C_{p,ot}) \) and that of newcomers \( (C_{p,nc}) \) \(^{19}\)

\[ C_{p,ot}^{p} - C_{p,nc}^{p} = \frac{\sigma}{1 - \xi} (\Omega_{t+1} - \gamma^{nc}) \]. (9)

where \( C_{p,t+1}^{o} \equiv \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} C_{p,t+1}(j) \), and \( C_{p,t+1}^{nc} \equiv C_{p,t+1}(t + 1) \). This mechanism, therefore, implies a direct channel through which financial instability can affect the real economy. An upward revision in financial-wealth expectations at time \( t \) implies that all active agents in \( t \) increase their current spending, so as to optimally smooth their intertemporal consumption profile; at \( t + 1 \), however, a fraction of these individuals will be replaced by newcomers that were previously inactive, and whose consumption at \( t + 1 \) will therefore not be affected by the current value

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16. I later discuss why this assumption, which considerably simplifies the analysis, is qualitatively inessential.

17. In particular, \( \mathcal{R}_t \equiv (1 - \xi)^{-1} - E_t \{ \mathcal{F}_{t,t+1}(1 + \pi_{t+1}) \} \).

18. Such “participation fee” serves the only purpose of implementing a steady-state cross-sectional distribution of financial wealth that is consistent with the social planner allocation. Please refer to the Online Appendix for details. This redistribution scheme, clearly, satisfies \( \sum_{j=-\infty}^{t} \xi(1 - \xi)^{t-j} \gamma(j) = 0 \).

19. Please refer to the Online Appendix for details.
of financial wealth. Thus, higher asset prices today, signaling an expected increase in future financial wealth, raise current consumption more than it does the expected discounted future level\textsuperscript{20}

Equation (9) shows that the magnitude of the wealth effect depends upon two factors. First, higher rates of replacement $\xi$, for given fluctuations in financial wealth, imply a larger heterogeneity among active agents. Second, stronger fluctuations in aggregate financial wealth translate a given heterogeneity among active agents into a larger wedge between the average consumption of old traders and that of newcomers. Moreover, it can be shown that this wedge distorts the decentralized equilibrium allocation with respect to the socially optimal one, in which the cross-sectional distribution of individual consumption is egalitarian and invariant to aggregate states\textsuperscript{21}. In this respect, equation (9) shows that, even if the economy starts from such social planner allocation, fluctuations in financial wealth induced by aggregate shocks make the decentralized cross-sectional consumption distribution endogenously deviate from the socially optimal one. This is at the heart of the normative implications that will be derived in the next sections. In particular, if either (i) there is no heterogeneity in market participants ($\xi = 0$) or (ii) aggregate financial wealth is stable at $\gamma^{nc}$, the wedge in average consumption between old traders and newcomers is closed, and the cross-sectional distribution of consumption is egalitarian.

The supply-side of the economy is standard New-Keynesian. A competitive retail sector packs a continuum of intermediate differentiated goods $Y_t(i)$ into a final good $Y_t$, using a CES technology. A monopolistic wholesale sector produces the former out of labor services, using the linear technology $Y_t(i) = A_t N_t(i)$, where $A_t$ is an aggregate productivity index, following $A_t = A_1^{1-\rho_a}A_{t-1}^{\rho_a} \exp(\epsilon_t^a)$, with $\rho_a \in [0, 1]$ and $\epsilon_t^a \sim N(0, \sigma_a^2)$. Aggregating across firms and using the equilibrium input demand coming from the retail sector yields $\exp(\Delta^p_t)Y_t = A_t N_t$, in which $N_t \equiv \int_0^1 N_t(i) \, di$ is the aggregate level of hours worked and

$$\Delta^p_t \equiv \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\mu} \, di$$

(10)
is a (second-order) index of price dispersion over the continuum of intermediate goods-producing firms, where $\varepsilon$ denotes the price-elasticity of demand for input $i$ and $\mu \equiv \varepsilon/(\varepsilon - 1)$ the corresponding degree of market power. As in most related literature, I assume that the government subsidizes employment at the constant rate $\tau$, to offset – in the steady state – the monopolistic distortion. Monopolistic firms are subject to the

\textsuperscript{20.} Equation (9) clarifies that, for a financial-wealth effect to arise, the assumption that “newcomers” enter with zero-holdings of financial assets is actually inessential. What is essential is that the average asset-holdings of “newcomers” differ from that of “old traders”. Since “newcomers” were previously inactive, this is generally the case, as they were not able to adjust it over time to smooth consumption in response to aggregate shocks, as done instead by “old traders”. Accordingly, a level of financial wealth that is passively rolled over while inactive – as for example assumed in Alvarez et al. (2002) – would also imply a financial-wealth effect of the same kind, while making analytical derivations considerably more cumbersome (since one would have to keep track of the distribution of wealth across “rule-of-thumbers” as well).

\textsuperscript{21.} Please refer to the Online Appendix for details.
“Calvo pricing”, with $1 - \theta$ denoting the probability for a firm of having the chance to re-optimize in a given period. When able to set its price optimally, each firm seeks to maximize the expected discounted stream of future dividends, otherwise they keep the price unchanged. Equilibrium in this side of the economy implies the familiar New-Keynesian Phillips Curve, relating current inflation to future expected inflation and current marginal costs.

Averaging across all agents in the economy yields the aggregate per-capita level of hours worked $N_t = \partial N_{p,t} + (1 - \theta) N_{r,t} = \partial N_{p,t} + (1 - \theta)/(1 + \delta)$ and consumption $C_t = \partial C_{p,t} + (1 - \theta)C_{r,t}$. The fiscal authority runs a balanced budget, financing the employment subsidies through lump-sum taxes to active agents: $\partial T_t = \tau W_t N_t$. In equilibrium, all output is absorbed by private consumption ($Y_t = C_t$), the net supply of state-contingent assets is nil, and the stock of outstanding equity shares is 1, for each monopolistic firm. As a consequence, aggregate profit income (enjoyed by active agents only) is: $\partial D_t = Y_t - (1 - \tau) N_t W_t$.

**2.2. The Optimal Steady State and the Linear Model**

In order to use a linear-quadratic approach to optimal policy analysis, here I define the “optimal steady state” as one that is consistent with the social planner allocation. In particular, the Online Appendix shows that the latter implies (in addition to aggregate efficiency) an egalitarian cross-sectional distribution of individual consumption that is invariant to aggregate states.

The decentralized allocation characterized in the previous section, thus, deviates from the socially optimal one because of three distortions. First, a static distortion due to monopolistic competition implies a lower level of supplied output, both in the short and in the long run. As usual, this distortion can be corrected by an appropriate employment subsidy $\tau$ such that $(1 + \mu)(1 - \tau) = 1$. Second, a dynamic distortion due to nominal rigidities implies a positively-sloped supply schedule, price dispersion across different firms and ultimately a lower level of short-run equilibrium output (this distortion fades away in a zero-inflation steady state). Third, an analogous dynamic distortion is due to the interaction in financial markets of heterogeneous agents, which implies that the cross-sectional consumption distribution across households changes endogenously in response to any aggregate shock affecting aggregate financial wealth, unlike in the social planner allocation. This third distortion is specific to the framework with financial-wealth effects and can be corrected in the steady state by an appropriate redistribution scheme $\gamma(j)$, for all $j$. To understand the analogy between the two dynamic distortions above, consider an economy starting from a steady state consistent with the social planner allocation, and it is hit by an aggregate shock. In the same way in which the aggregate shock triggers an inefficient response of the relative-price distribution across heterogeneous (in terms of price duration) firms, it also triggers an analogously inefficient response of the cross-sectional consumption distribution across heterogeneous (in terms of asset holdings) consumers: social welfare is reduced by
endogenous consumption dispersion in an analogous way in which it is by endogenous price dispersion.

A socially optimal steady state can be implemented by a redistribution scheme γ(j), such that, for all j, γ(j) = Ω(j) − (1 − ξ)Ω. As a consequence, the steady-state interest rate satisfies the familiar β(1 + r) = 1, and all agents supply the same amount of hours worked Np(j) = Nr = N = (1 + δ)−1 and consume the same level of consumption Cp(j) = Cr = C = Y = A(1 + δ)−1, for all j.

First-order approximation around such optimal steady state yields:

\[ y_t = E_t y_{t+1} + \frac{\psi}{\Theta} E_t \omega_{t+1} - \frac{1}{\Theta} (r_t - E_t \pi_{t+1} - \rho) + \frac{1 - \Theta}{\Theta} E_t \Delta a_{t+1} \]  
(11)

\[ \omega_t = \beta E_t \omega_{t+1} - (1 - \beta) \frac{1 + \varphi - \mu}{\mu} (y_t - a_t) - \beta (r_t - E_t \pi_{t+1} - \rho) + (1 - \beta) a_t \]  
(12)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - a_t). \]  
(13)

where φ ≡ N/(1 − N) = 1/δ denotes the steady-state labor-leisure ratio, also capturing the elasticity of the real wage with respect to aggregate hours worked, κ ≡ \( \theta^{-1}(1 - \theta)(1 - \theta\beta)(1 + \varphi) \) is the slope of the Phillips Curve, \( \rho \) the steady-state real interest rate, and \( \psi/\Theta \) the elasticity of output with respect to expected future financial wealth, where

\[ \psi \equiv \xi \frac{1 - \beta(1 - \xi)}{(1 - \xi)(1 + \delta)} \frac{1}{1 - \beta} \frac{\mu}{1 + \mu} \geq 0 \]

and \( \Theta \equiv 1 - \varphi(1 - \theta)/\theta \).

The partition of consumption between active and inactive agents is finally determined by

\[ c_{p,t} = \Theta y_t + (1 - \Theta) a_t \]  
(14)

\[ c_{r,t} = (1 + \varphi) y_t - \varphi a_t. \]  
(15)

2.2.1. The Flexible-Price and Socially Optimal Allocations. The flexible-price equilibrium (FPE) arises in this model when the probability of having to charge last period’s price goes to zero (θ = 0). In this case, denoting variables in the FPE with an upper bar, real marginal costs are constant and the optimal employment subsidy implements the efficiency condition \( \bar{W}_t = A_t \). Accordingly, labor-market

22. One notable difference is that the former dispersion only affects the distributional implications of the equilibrium allocation, while the latter affects also aggregate efficiency.

23. Let \( z_t \equiv \log(Z_t/Z) \) for \( z = y, \omega, a; r_t \) is the net nominal interest rate and \( \pi_t \) the net inflation rate.

24. As shown by Bilbiie (2008), \( \Theta \) relates the average participation rate to financial markets \( \theta \) to the slope of the forward-looking IS curve, with the threshold \( \theta^* = \varphi/(1 + \varphi) \) determining whether the economy behaves according to the standard (\( \theta > \theta^* \)) or the inverted (\( \theta < \theta^* \)) aggregate demand logic. In the rest of the analysis I will restrict attention to the case (\( \theta > \theta^* \)).
clearing implies the equilibrium level of potential output: \( \tilde{y}_t = a_t \), and the equilibrium allocation can be derived as the solution to the system:

\[
\begin{align*}
\tilde{y}_t &= a_t \quad (16) \\
\tilde{\pi}_t &= 0 \quad (17) \\
\tilde{r}_t &= \rho + E_t \Delta a_{t+1} + \psi E_t \tilde{\omega}_{t+1} \quad (18) \\
\tilde{\omega}_t &= \beta E_t \tilde{\omega}_{t+1} - \beta (\tilde{r}_t - \rho) + (1 - \beta) a_t. \quad (19)
\end{align*}
\]

Thus, the FPE level of financial wealth is proportional to aggregate productivity

\[
\tilde{\omega}_t = \frac{1 - \beta \rho_a}{1 - \beta \rho_a (1 - \psi)} a_t, \quad (20)
\]

and the real interest rate supporting such allocation is

\[
\tilde{r}_t = \rho + E_t \Delta a_{t+1} + \frac{1 - \beta \rho_a}{1 - \beta \rho_a (1 - \psi)} \psi \rho_a a_t. \quad (21)
\]

This allocation features aggregate efficiency because nominal rigidities are shut down and the static distortion related to monopolistic competition is corrected by the employment subsidy. It is not, however, consistent with a cross-sectional consumption distribution that is invariant across aggregate states, as instead implied by the socially optimal allocation. Indeed, while the heterogeneity between the sets of active and inactive agents fades away, as implied by \( (14) - (16) \), the heterogeneity within the set of market participants does not, and in fact responds to fluctuations in financial wealth, which still affect the wedge in average consumption between newcomers and old traders. As a result, the flexible-price allocation is suboptimal with respect to the socially optimal one. With the benchmark specification of the model (in which \( \xi = 0 \)) the DAMP economy shares the same natural level of output, but the natural levels of real interest rate and financial wealth are generally different. In the benchmark setups, the natural rate of interest fully accommodates the pressures on output coming from productivity shocks (\( \tilde{r}_t = \rho + E_t \Delta a_{t+1} \)), and financial wealth mirrors the evolution of productivity (\( \tilde{\omega}_t = a_t \)). Equation \( (21) \), instead, shows that in the DAMP economy the natural interest rate leans partially against productivity shocks, to absorb the fluctuations in financial wealth that would otherwise distort the dynamics of aggregate consumption.

The socially optimal allocation – characterized by both aggregate efficiency and optimal cross-sectional consumption distribution – can be defined as an equilibrium in which not only the output gap and inflation rate are zero, but also financial wealth is

\[
(25). \quad \text{Since in the flexible-price equilibrium the Phillips Curve is vertical, the equilibrium level of real activity is determined by the supply side of the economy only, for which fluctuations in financial wealth are neutral.}
\]

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stable at the steady-state level \(\omega_t^e = 0\):

\[
\begin{align*}
\gamma_t^e &= \bar{y}_t = a_t \\
\pi_t^e &= 0 \\
\omega_t^e &= 0.
\end{align*}
\]

As implied by equation (9), when financial wealth is stable at \(\gamma^{nc}\), the cross-sectional consumption dispersion is zero, and the distributional efficiency of the equilibrium allocation is restored. However, unlike in the FPE, there is no real interest-rate path that supports this equilibrium allocation.

**Proposition 1.** In a decentralized DAMP economy with an active stock market, there is no real interest rate dynamics supporting the socially optimal allocation.

*Proof.* Impose \(y_t = a_t\) and \(\pi_t = \omega_t = 0\) on system (11)–(13). Then equation (11) implies \(\pi_t^e = \rho + E_t \Delta a_{t+1}\), while equation (12) implies \(\pi_t^e = \rho + [(1 - \beta)/\beta]a_t\), which are the same only as long as \(a_t = 0\) for all \(t\).

2.3. The Welfare-Based Loss Function.

Given Proposition 1, it becomes useful to evaluate the welfare implications of targeting the FPE allocation instead of the socially optimal one. To this aim, I now derive the monetary-policy loss function as a second-order approximation of social welfare around the optimal steady state.

Define social welfare at \(t_0\) as

\[
\mathcal{W}_{t_0} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \partial \mathcal{U}_{p,t} + (1 - \theta) \mathcal{U}_{r,t} \right] \right\},
\]

where \(\mathcal{U}_{p,t} = \sum_{j=-\infty}^{t} \xi (1 - \xi)^{t-j} U_{p,t}(j)\) is the average period utility of active agents, \(\mathcal{U}_{r,t} = \sum_{k=-\infty}^{t} \phi (1 - \phi)^{t-k} U_{r,t}(k) = U_{r,t}\) that of “rule-of-thumbers”, and \(U_{s,t}(h)\) is defined in (3), for \(s = p, r\) and \(h = j, k\).

Given the efficiency condition \(\varphi \delta = 1\) implied by the optimal employment subsidy, and ignoring terms independent of policy and of higher order, a second-order Taylor approximation of \(\mathcal{W}_{t_0}\) is

\[
\mathcal{W}_{t_0} \approx E_{t_0} \left\{ \beta^{t-t_0} \left[ \partial \mathcal{U}_{p,t} + (1 - \theta) \mathcal{U}_{r,t} \right] \right\}.
\]

26. This approach satisfies the conditions that deliver a valid second-order approximation of expected welfare when evaluated using private-sector equilibrium conditions that are approximated only to first order. See Woodford (2003).

27. The definitions of \(\mathcal{U}_{p,t}\) and \(\mathcal{U}_{r,t}\) are consistent with the assumption that different longevities are treated equally by the social planner. Please refer to the Online Appendix for details. Analogous definitions of social welfare are adopted by Curdia and Woodford (2010), in a model with Markov-Switching agent-types, and by Bilbiie (2008), among others, in the benchmark LAMP framework.
expansion of the aggregate period—\( t \) utility for active agents can be written as

\[
U_{p,t} = c_{p,t} - \frac{1}{2} \text{var}_j c_{p,t}(j) - n_{p,t} + \frac{1}{2} \text{var}_j n_{p,t}(j) - \frac{1 + \varphi}{2} E_j \left( n_{p,t}(j)^2 \right)
\]

\[
= c_{p,t} - n_{p,t} - \frac{1 + \varphi}{2} n_{p,t}^2 - \frac{1 + \varphi}{2\varphi} \text{var}_j c_{p,t}(j).
\]

(26)

in which the last equality uses \( E(z^2) = E(z)^2 + \text{var}(z) \) and a first-order approximation of the individual labor supply, to relate the cross-sectional variance of hours worked to that of consumption: \( \varphi^2 \text{var}_j n_{p,t}(j) = \text{var}_j c_{p,t}(j) \). Since inactive agents are homogeneous and supply a constant amount of hours worked, a second-order approximation of their period—\( t \) utility is simply \( U_{r,t} = c_{r,t} \). Using the resource constraint and a second-order approximation of the aggregate production function, one then obtains the period—\( t \) social utility as a function of output gap \( x_t \), price dispersion across firms \( \Delta^p \) and consumption dispersion across active agents \( \Delta^c_{p,t} \equiv \text{var}_j c_{p,t}(j) \):

\[
\partial U_{p,t} + (1 - \partial) U_{r,t} = -\left( \frac{1 + \varphi}{2\varphi} \right) \left( x_t^2 + \frac{2\partial}{1 + \varphi} \Delta^p + \frac{\varphi^2}{\varphi} \Delta^c_{p,t} \right).
\]

(27)

Social welfare thus displays an additional second-order term, compared to the LAMP framework, which is related to the cross-sectional dispersion in consumption \( \Delta^c_{p,t} \): were all active agents homogenous, consuming in every period the same amount of goods, the cross-sectional variance would be zero, and the LAMP model would arise. To understand the implications of this additional term, it is useful to recall the partition of the set of active agents in “newcomers” and “old traders”. Indeed, using the law of total variance on such partition, one can characterize the evolution over time of the cross-sectional consumption dispersion across all active agents in the economy as:

\[
\Delta^c_{p,t} = (1 - \xi) \Delta^c_{p,t-1} + \psi \frac{\sigma}{1 - \beta} \frac{\mu}{1 + \mu} \omega_t^2.
\]

(28)

Moving from any arbitrary initial level \( \Delta^c_{p,t_0-1} \), which is independent of policies implemented from \( t = t_0 \) onward, I can therefore write the discounted value over all periods \( t > t_0 \) as

\[
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta^c_{p,t} = \frac{\psi \mu}{(1 + \delta)(1 - \beta)(1 + \mu)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \omega_t^2
\]

(29)

Finally, the time—\( t_0 \) conditional expectation of the discounted stream of future period social losses yields the welfare-based loss function, as a share of steady-state aggregate consumption:

\[
L_{t_0} = -\mathcal{W}_{t_0} = \frac{1 + \varphi}{2\varphi} E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( x_t^2 + \alpha_p \pi_t^2 + \alpha_w \omega_t^2 \right) \right\}.
\]

(30)

28. For details on the derivation, please refer to the Online Appendix.

29. The second equality in \( \sum_{t=t_0}^{\infty} \beta^{t-t_0} \text{var}_j p_{t}(i) = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \pi_t^2 \).
In the loss function above, \( \alpha_{\pi} \equiv \theta \varepsilon / \kappa \) denotes the relative weight on inflation stability and \( \alpha_{\omega} \equiv \theta^2 \psi \mu / [(1 + \varphi)(1 - \beta)(1 + \mu)] \geq 0 \) the one on financial stability. As discussed in Bilbiie (2008), the relative weight on inflation is a fraction \( \theta \) of that arising in the standard Ricardian model – i.e. the ratio between the price-elasticity of demand \( \varepsilon \) and the slope of the Phillips Curve \( \kappa \) – since inflation variability reduces profit income, which is only enjoyed by market participants. The additional weight on financial stability is instead peculiar to the DAMP framework, reflecting the fact that fluctuations in aggregate financial wealth affect the cross-sectional consumption distribution across active agents and make it deviate from the socially optimal one.

3. Stabilization Trade-offs and the Optimal Policy

The loss function (30) shows that cross-sectional consumption dispersion is induced by financial instability and is a source of welfare loss, and implies that the flexible-price allocation is therefore not a proper target for welfare-maximizing monetary policy.

**Proposition 2.** In a decentralized DAMP economy with an active stock market, a welfare-maximizing policy maker should target the socially optimal allocation. Financial stability \( (\omega_t = 0) \) becomes an explicit target of welfare-maximizing monetary policy, in addition to output and inflation stabilization \( (x_t = \pi_t = 0) \). The welfare-relevance of financial stability \( (\alpha_{\omega}) \) is higher when financial-wealth effects \( (\psi) \) are stronger, the market power of monopolistic firms \( (\mu) \) is higher and/or the average asset-market participation rate \( (\theta) \) is higher.

An interesting implication of Propositions 1 and 2 is that a welfare-relevant stabilization tradeoff endogenously arises, even absent inefficient supply shocks: since there is no interest-rate dynamics supporting the socially optimal allocation, minimization of the loss function (30) implies that some fluctuations in inflation and the output gap will be optimal in order to reduce financial instability.

We can see this formally, by evaluating optimal policy in this framework along two dimensions: the optimal response of the DAMP economy to a productivity shock, and the expected welfare loss implied by a specific policy regime, captured by the unconditional mean of the period loss in (30):

\[
E\{-U_t\} = \frac{(1 + \varphi)}{2 \theta} \left( \text{var}(x_t) + \alpha_{\pi} \text{var}(\pi_t) + \alpha_{\omega} \text{var}(\omega_t) \right). \tag{31}
\]

The complete system of equilibrium conditions for the private sector, constraining the optimal policy, can be expressed in terms of the relevant gaps from the socially

---

30. Notice that an analogous (and even stronger) role for financial stability arises in the welfare criterion of a perpetual-youth economy, which is nested in the DAMP framework under the parameterization \( \theta = 1 \).
optimal allocation:

\[
x_t = E_t x_{t+1} + \frac{\psi}{\theta} E_t \omega_{t+1} - \frac{1}{\Theta} (r_t - E_t \pi_{t+1} - \tilde{r} \pi_{t+1})
\]

(32)

\[
\omega_t = \beta E_t \omega_{t+1} - (1 - \beta) \frac{1 + \varphi - \mu}{\mu} \pi_t - \beta (r_t - E_t \pi_{t+1} - \tilde{r} \pi_{t+1}) + (1 - \beta \rho_a) a_t
\]

(33)

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,
\]

(34)

in which \( \tilde{r} \pi_{t+1} \equiv \rho + E_t \Delta a_{t+1} \). As usual in this class of models, I can determine the optimal path of the nominal interest rate residually, and derive first the optimal allocation in terms of the other endogenous variables. In this case, the constraints that are relevant for optimal policy can be given the following reduced form:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,
\]

(35)

\[
\omega_t = \beta (1 - \psi) E_t \omega_{t+1} + \eta x_t - \beta \Theta E_t x_{t+1} + (1 - \beta \rho_a) a_t
\]

(36)

in which \( \eta \equiv [\mu - (1 - \beta)(1 + \varphi)] \mu^{-1} - \beta (1 - \Theta) \). The IS schedule (32) – or equation (33) – can then be used to back out the equilibrium path of the nominal interest rate consistent with the optimal policy.

To understand the tradeoffs arising in this economy, consider a transitory productivity shock \( \rho_a = 0 \) and solve equations (35) and (36) forward:

\[
\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k},
\]

(37)

\[
\eta x_t = \omega_t + \left(1 - \frac{\eta (1 - \psi)}{\Theta}\right) \sum_{k=1}^{\infty} \left(\frac{\beta \Theta}{\eta}\right)^k E_t \omega_{t+k} - a_t.
\]

(38)

Equation (37) shows the familiar result that inflation depends on the current as well as expected future output gaps. Absent inefficient supply shocks, this equation clarifies that targeting the flexible-price allocation at all times is consistent with price stability, regardless of productivity shocks: no tradeoff exists between inflation and output-gap. Equation (38), in contrast, implies that a stabilization tradeoff does exist between real and financial stability: conditional on a positive productivity shock, the economy will necessarily experience either a fall in the output gap (and thereby also inflation), or an increase in financial wealth. In the standard Ricardian economy (or the benchmark LAMP model), these tradeoffs are meaningless from a welfare perspective because \( \omega_t \) does not enter the loss function, and are trivially resolved because financial stability is not a concern for the policy maker. In a DAMP economy, instead, \( \omega_t \) is an explicit argument of the welfare criterion, and these tradeoffs therefore require explicit consideration from welfare-maximizing policy makers. The forward-looking nature of inflation and asset prices, moreover, suggests that such tradeoffs can be improved by credibly committing to a specific state-contingent path for financial wealth and/or the output gap, so that the central bank can reduce the welfare losses associated to any shock.
3.1. Optimal Policy under Discretion and Full Commitment

One way of dealing with the above tradeoffs is for the central bank to choose in every period the optimal level of all target variables, so as to minimize the period welfare loss. The optimal policy under discretion therefore minimizes the period-loss function

$$ \frac{(1 + \varphi)}{2\theta} \left( x_t^2 + \alpha_{\pi} \pi_t^2 + \alpha_{\omega} \omega_t^2 \right) $$

such that $\omega_t = \eta x_t + K_{\omega,t}$ and $\pi_t = \kappa x_t + K_{\pi,t}$, where $K_{\pi,t}$ and $K_{\omega,t}$ collect the terms related to expectations about the future and exogenous state variables, that the central bank cannot affect in this policy regime. The solution to this problem requires the following optimal targeting rule:

$$ x_t + \alpha_{\pi} \kappa \pi_t + \alpha_{\omega} \eta \omega_t = 0. \tag{39} $$

Using the above relation in the reduced system (35)–(36), yields the optimal state-contingent path for the inflation rate, output gap and financial wealth:

$$ \pi_t^d = -\Phi_{\pi} a_t \tag{40} $$

$$ x_t^d = -\Phi_{x} a_t \tag{41} $$

$$ \omega_t^d = \Phi_{\omega} a_t, \tag{42} $$

where

$$ \Phi_{\pi} \equiv \frac{\alpha_{\omega} \kappa \eta (1 - \beta_{p_a})}{\mathcal{A} + \alpha_{\omega} \mathcal{B} + \psi \mathcal{C}}, \quad \Phi_{x} \equiv \frac{\alpha_{\omega} \eta (1 - \beta_{p_a})^2}{\mathcal{A} + \alpha_{\omega} \mathcal{B} + \psi \mathcal{C}}, \quad \Phi_{\omega} \equiv \frac{\mathcal{A}}{\mathcal{A} + \alpha_{\omega} \mathcal{B} + \psi \mathcal{C}}, $$

with $\Phi_{\omega} \in [0, 1], \mathcal{A} \equiv (1 - \beta_{p_a})(1 + \alpha_{\pi} \kappa^2 - \beta_{p_a})$, $\mathcal{B} \equiv (1 - \beta_{p_a})(\eta - \Theta \beta_{p_a}) \eta$ and $\mathcal{C} \equiv \beta_{p_a}(1 + \alpha_{\pi} \kappa^2 - \beta_{p_a})$.

In the case with no real effects of financial instability ($\xi = \psi = \alpha_{\omega} = 0$) – regardless of the share of active agents $\theta$ – optimal monetary policy under discretion requires price stability ($\Phi_{\pi} = 0$), output-gap stability ($\Phi_{x} = 0$) and that all volatility be borne by financial wealth ($\Phi_{\omega} = 1$). This is the familiar case of the optimality of price stability in an economy with no inefficient supply shocks, in which the flexible-price allocation is efficient. The picture changes in a DAMP economy:

**Proposition 3.** In a decentralized DAMP economy with an active stock market, price stability is not an optimal policy regime from a welfare perspective.

Indeed, given convexity of the welfare-based loss function, the optimal discretionary monetary policy requires the volatility implied by productivity shocks to be shared among the three targets. Accordingly, following a positive productivity shock, financial wealth will rise less than under price stability ($\Phi_{\omega} < 1$), and both inflation and the output gap will decline ($\Phi_{\pi}, \Phi_{x} < 1$).

An alternative way of dealing with the stabilization tradeoff is to commit to a state-contingent path for the target variables. If the commitment is credible, there is room...
for improving the tradeoff by anchoring expectations, as in the standard framework. In this case, the monetary policy problem at time $t_0$ is to minimize the loss function (30) such that the constraints (35)–(36) are satisfied for all $t$. The optimality conditions for this policy problem under full commitment are:

$$x_t = \eta \lambda_{2,t} - \Theta \lambda_{2,t-1} + \kappa \lambda_{1,t}$$  \hspace{1cm} (43)

$$\alpha_r \pi_t = \lambda_{1,t-1} - \lambda_{1,t}$$  \hspace{1cm} (44)

$$\alpha_\omega \omega_t = (1 - \psi) \lambda_{2,t-1} - \lambda_{2,t}.$$  \hspace{1cm} (45)

in which $\lambda_{1,t}$ and $\lambda_{2,t}$ are the Lagrange multipliers associated to constraints (35)–(36), respectively, and $\lambda_{1,t0-1} = \lambda_{2,t0-1} = 0$. In the benchmark LAMP case with $\xi = \alpha_\omega = 0$, equations (43)–(45) imply $\lambda_{2,t} = 0$ for all $t$, and therefore price stability is still the optimal policy. In a DAMP economy, instead, $\lambda_{2,t}$ will generally differ from zero, which makes the trade-off discussed earlier relevant for output and inflation, and implies that Proposition 3 applies also under full commitment.

Figure 1 provides a graphical illustration of such proposition, and shows an impulse-response simulation exercise to display the policy implications for the DAMP economy. The figure shows the dynamic response of selected variables – under alternative policy regimes – to a transitory, one-standard-deviation productivity shock, with standard deviation $\sigma_a = 0.5\%$. Table 1 then compares the cyclic and welfare implications of the alternative policy regimes, reporting the equilibrium standard deviations (in percentage points) of selected variables and the implied welfare losses (in basis points of steady-state aggregate consumption).

In the numerical illustration, the key parameters to calibrate are the share of savers $\vartheta$ and the turnover rate $\xi$. As a baseline calibration, I use $\vartheta = 0.8$ and $\xi = 0.17$. The former is the estimated value reported by Bilbiie and Straub (2013) for the Great-Moderation sample (i.e. post 1984); the latter is chosen to imply an elasticity of aggregate consumption to stock-market wealth ($\psi/\Theta$) of about 0.15, as documented by Ludvigson and Steindel (1999) for the Great-Moderation sample, and is consistent with the turnover rate in U.S. financial portfolios reported by Cella et al (2013), which implies an average investment horizon of about 6 quarters.

The circled line in the Figure 1 displays the case of Inflation Targeting, under which both inflation and the output gap can be fully stabilized – given the absence of inefficient supply shocks – and all the volatility induced by the productivity shock

---

31. Please refer to the Online Appendix for an extensive sensitivity analysis of the main results to the calibration of these key parameters. The baseline calibration $\xi = 0.17$ is also consistent with the replacement rate and the financial-wealth effects documented by Castelnuovo and Nisticò (2010) within an empirical version of the perpetual-youth specification of this model, estimated with post-WWII US data. The rest of the model is calibrated using convention or previous studies: the time discount factor is $\beta = 0.99$, to imply a steady-state annualized real interest rate of 4%, the Calvo parameter $\theta = 0.75$, implying an average price duration of 4 quarters, the average price-markup is $\mu = 20\%$, consistently with Bilbiie and Straub (2013), among others, and the elasticity of real wages to aggregate hours is $\phi \equiv N/(1 - N) = 0.3$, consistently with Rotemberg and Woodford (1997), Gali et al (2007), and with the average hours worked for the US in the Great-Moderation sample.
is borne by financial wealth and the interest rate. These dynamics are the same as those that would arise in the standard Ricardian economy – in which $\xi = 0$ and $\vartheta = 1$ – and in the benchmark LAMP framework – in which $\xi = 0$ and $\vartheta \in (0, 1)$]32. However, in those frameworks fluctuations in financial wealth do not have distributional effects and the welfare loss is therefore zero (see first column of Table 1). In the DAMP economy, instead, fluctuations in financial wealth do affect the cross-sectional consumption dispersion among savers $\Delta \xi_c$ – evolving according to (28) – which in this case is detrimental from a welfare perspective. The baseline calibration, indeed, implies a weight on financial stability of the same order of magnitude as that on the output gap: $\alpha_\omega = 1.18$. As a result, under Inflation Targeting, the welfare loss implied by a productivity shock with standard deviation of 0.5% – evaluated using equation (31) – is about 0.239bp of steady-state aggregate consumption, as reported in the second column of Table 1.

The dashed line in Figure 1 shows instead the dynamic response of the DAMP economy under the optimal discretionary policy. In this case, equations (40)–(42) implies that monetary policy finds it optimal to cut inflation by about 10bp and output gap by about 20bp, in order to reduce by about 35% the short-run response of financial wealth to the shock. This requires cutting the nominal interest rate about 75bp less

32. Indeed, equations [20] and [21] show that, when $\rho_\alpha = 0$, Inflation Targeting implies the same path for interest rates and financial wealth as in the RA setup.

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**Figure 1.** Response of selected variables to a transitory, one-standard-deviation productivity shock. Circled line: Inflation Targeting. Dashed line: optimal policy (Discretion). Solid line: optimal policy (Full Commitment).
Table 1. Implied volatilities and welfare losses conditional on a productivity shock.

<table>
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<tr>
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<th>Benchmark</th>
<th>DAMP</th>
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<tr>
<td></td>
<td>IT</td>
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</tr>
<tr>
<td></td>
<td>Disc</td>
<td>Comm</td>
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<td>Inflation</td>
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<tr>
<td>Output Gap</td>
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<td>0.152</td>
<td>0.138</td>
</tr>
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</table>

Note: Implied volatilities are standard deviations in percentage points; Welfare Losses are bases points of steady-state aggregate consumption. Standard deviation of productivity shock is 0.5%. IT: Inflation Targeting; Disc: Discretion; Comm: Full Commitment.

than under Inflation Targeting, and allows to reduce the response of cross-sectional consumption dispersion by over a half, throughout the transition. From the cyclical perspective, as reported by the third column of Table 1, the effect is that the volatility implied by the productivity shock is now “shared” among the three welfare-relevant variables: the standard deviation of inflation and the output-gap raises from zero to, respectively, 0.024% and 0.210%, while that of financial wealth drops from 0.5% to 0.32%. Given convexity of the welfare criterion, this reduces the welfare loss to about 0.152bp of steady-state aggregate consumption: price stability in this environment is substantially suboptimal, as it implies a welfare loss about 57% larger than under the optimal discretionary policy.

The general implication is reinforced under full commitment, as implied by the solid line in Figure 1 and the fourth column of Table 1. As to the dynamic response of the economy, the commitment takes two forms. On the one hand monetary policy commits to keep financial wealth higher than steady state for some time even after the shock has reverted back to mean, through persistently lower nominal interest rates (which on impact fall more than 1 percentage point less than under Inflation Targeting); this achieves a smaller response of financial wealth on impact and a lower consumption dispersion throughout the transition. On the other hand, it also commits to an output boom after the shock has returned to steady state, in order to dampen the short-run effect of the shock on the inflation rate, which now falls by about 6bp only. Interestingly, note that the short-run response of the output gap in this case is slightly stronger than under discretion: the tradeoff between financial and real variables is such that even full commitment to a specific state-contingent path for the target variables is not enough to dampen the short-run effects of the shock on all of them, unlike in the standard analysis of commitment with inefficient supply shocks. From the cyclical perspective, exploiting the effect of the credible commitment on expectations enables the central bank to further reduce the welfare loss to 0.138bp of steady-state aggregate consumption.

Overall, therefore, the optimal response of the DAMP economy – whether under discretion or commitment – requires non-negligible deviations from the corresponding
response under the Inflation Targeting regime, that would instead be optimal in an economy with no financial-wealth effects (whether Ricardian or LAMP). In particular, Inflation Targeting implies welfare losses up to 73% higher than under optimal policy, and a variability of consumption dispersion up to three times higher (as implied by the fifth row of Table 1).

3.2. Optimal Policy with Inefficient Supply Shocks

Until now I have considered only productivity shocks to show that a meaningful tradeoff for monetary policy arises in a DAMP economy even when there are no inefficient supply shocks that break the link between inflation and the welfare-relevant output gap. Here I extend the analysis to the case in which such shocks exist, to show – perhaps not surprisingly, at this point – that also in the presence of the familiar inflation-output tradeoff, ignoring the role of financial-wealth effects (whether for welfare or for the propagation mechanism) may have non-trivial implications.

In particular, consider an environment in which the market power of monopolistic firms ($\mu$) is subject to exogenous stochastic disturbances. In this case, the Phillips Curve is subject to a cost-push shock $u_t$, capturing the stochastic deviations of flexible-price output from potential one:

$$\pi_t = \beta E_{t} \pi_{t+1} + \kappa x_t + u_t,$$

where cost-push shocks follow $u_t = \rho_u u_{t-1} + \varepsilon^u_t$, with $\rho_u \in [0, 1]$ and $\varepsilon^u_t \sim N(0, \sigma_u^2)$.

Also in this case, the targeting rules (39) and (43)–(45) characterize the optimal policy under discretion and commitment, respectively. The welfare losses implied by alternative policy regimes, however, now depends on the cyclical features of both shocks.

Consider, as before, purely transitory shocks: a productivity shock $a_t$ with standard deviation $\sigma_a = 0.5\%$, and a cost-push shock $u_t$ with standard deviation $\sigma_u = 0.08\%$. I first evaluate the relevance of financial-wealth effects for the monetary-policy loss function, by comparing the dynamic and cyclical implications of four alternative policy regimes. The first two regimes are the optimal policy under discretion and commitment, respectively. The other two regimes consider instead the case in which the central bank disregards the welfare implications of financial-wealth effects and therefore adopts the loss function that would arise in the standard LAMP setup, as in Bilbiie (2008):

$$L_{t_0} = \frac{(1 + \varphi)}{2\theta} E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( x_t^2 + \alpha \pi_t^2 \right) \right\},$$

33. The standard deviations of productivity and cost-push shocks are calibrated such that the cyclical implications of the model are consistent with empirical evidence along two dimensions: i) cost-push shocks are about 15% as important as productivity shocks in explaining the variance of real output, and ii) the standard deviation of the cross-sectional variance of log-consumption is about 0.02 (consistently with Krueger and Perri, 2006). Consistency with the evidence above is evaluated with respect to a policy regime in which the central bank follows a standard Taylor Rule with response coefficients to inflation and output gap equal to 1.5 and 0.5, respectively.
with $\alpha_\pi \equiv \vartheta \varepsilon / \kappa$, which is maximized under either discretion or commitment. I will refer to this case as “Flexible Inflation Targeting” regime, consistently with Svensson (1999). Figure 2 displays the optimal response of the economy to a one standard-deviation cost-push shock under the four regimes, and the first four columns of Table 2 report their cyclical and welfare implications.

Ignoring the welfare role of financial instability by minimizing loss (47) instead of (30) induces non-negligible differences both in the short-run response of the economy to a cost-push shock and in the implied welfare losses. In particular, the monetary policy response is overly restrictive, implying higher nominal interest rates (in the order of 20 to 30bp) and lower output gap (in the order of 7 to 10bp) and inflation (between 3 and 5bp), while consumption dispersion increases by about twice as much, under both discretion and commitment. As a result, the welfare loss implied by ignoring the welfare relevance of financial instability in the DAMP economy is about 31% higher in the case of discretion, and about 41% higher in the case of commitment, as implied by Table 2.

A second dimension along which the relevance of financial-wealth effects can be evaluated is their role in the propagation of structural shocks with respect to the LAMP benchmark. Indeed, comparing the loss under optimal policy in the DAMP model (first two columns of Table 2) with the corresponding loss in the LAMP framework (i.e. $\xi = 0$, last two columns) shows that the welfare costs associated to the additional
Table 2. Implied volatilities and welfare losses conditional on productivity and cost-push shocks.

<table>
<thead>
<tr>
<th></th>
<th>DAMP</th>
<th>Flexible IT</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Disc Comm</td>
<td>Disc Comm</td>
<td>Disc Comm</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.067 0.055</td>
<td>0.052 0.045</td>
<td>0.052 0.045</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.264 0.284</td>
<td>0.250 0.216</td>
<td>0.250 0.216</td>
</tr>
<tr>
<td>Financial Wealth</td>
<td>0.347 0.310</td>
<td>0.544 0.523</td>
<td>0.544 0.529</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.339 0.235</td>
<td>0.551 0.505</td>
<td>0.551 0.506</td>
</tr>
<tr>
<td>Cons. Dispersion</td>
<td>0.030 0.025</td>
<td>0.073 0.068</td>
<td>0.000 0.000</td>
</tr>
<tr>
<td>Welfare Loss</td>
<td>0.326 0.263</td>
<td>0.429 0.371</td>
<td>0.146 0.110</td>
</tr>
</tbody>
</table>

Note: implied volatilities are standard deviations in percentage points; Welfare Losses are basis points of steady-state aggregate consumption. Standard deviation of productivity shocks is 0.5%; standard deviations of cost-push shocks is 0.08%. Flexible IT: Flexible Inflation Targeting; Disc: Discretion; Comm: Full Commitment.

distortion characterizing the DAMP economy are substantial, as they imply welfare losses more than twice as high, under both discretion and commitment.

Therefore, the total welfare costs of ignoring the financial-wealth effects reflect both dimensions analyzed above – their role in the propagation of structural shocks and in the relevant welfare criterion – and can be evaluated by comparing the welfare loss implied by the Flexible Inflation Targeting regime in the DAMP economy (third and fourth columns in Table 2) with the loss under optimal policy in the benchmark LAMP framework (last two columns): a policy maker that believes to be operating in an economy where financial developments are neutral for equilibrium allocation when, in fact, they are not, induces, on average, welfare losses that are about 3 times higher than expected, under both discretion and commitment.

Overall, the quantitative relevance of these welfare implications – though they may seem small in absolute terms – is rather sizeable in relative terms, and suggests potentially important implications of ignoring the financial-wealth channel in the design of monetary policy.\footnote{Also in absolute terms, the reported welfare implications are in the same order of magnitude that characterizes welfare exercises of this kind that have been used in the literature to rank alternative policy regimes. See, among others, Galí and Monacelli (2005). Please refer to the Online Appendix for a sensitivity analysis of the implications above.}

4. Conclusion

This paper presents a dynamic stochastic general equilibrium model of an economy where agents participate to asset markets only discontinuously. The framework implies two layers of agents heterogeneity: between types (market participants versus rule-of-thumbers) and within the type of market participants (because of different longevities in the type). This latter kind of heterogeneity implies financial-wealth effects on aggregate consumption and makes all agents non-Ricardian.
The *discontinuous asset market participation* (DAMP) model nests as special cases most frameworks currently used to study monetary policy: (i) the standard representative-agent model, where all agents are infinitely-lived, homogeneous, and with continuous access to asset markets (hence Ricardian); (ii) the LAMP model, where all agents are infinitely-lived and heterogeneous only *between* groups, as some have continuous access to asset markets (Ricardian) and some have no access at all (non-Ricardian); (iii) the perpetual-youth model, where all agents have continuous access to asset markets but they are finitely-lived, and therefore heterogeneous (and non-Ricardian). With respect to the latter framework, one appealing feature of the DAMP model is that it implies the same kind of wealth-effects on aggregate consumption, but within an infinitely-lived-agent economy: the replacement rate in asset markets is fundamentally disconnected from expected lifetimes. The exogenous transition between agent types captures, in reduced form, the effects of occasionally-binding borrowing (or liquidity) constraints induced by different kinds of idiosyncratic uncertainty: the DAMP model is thus flexible enough to address different issues related to agents’ heterogeneity, where the specific structural interpretation is reflected in the calibration of the relevant transition probabilities.

Within a DAMP economy with an active stock market – where the transition between agent types is interpreted as a reduced-form representation of the interaction between a costly decision to participate in asset markets and idiosyncratic uncertainty about individual income – the paper then provides a formal normative analysis of monetary policy, deriving a welfare-based monetary-policy loss function and characterizing the interplay between optimal policy and financial stability.

A second contribution is the derivation of the welfare criterion *per se*: since the demand side of the economy features an infinite number of heterogenous agents, it implies aggregation issues, when deriving a second-order approximation of social welfare. These issues arise also in standard perpetual-youth models, and are behind the lack of optimal policy analyses in a linear-quadratic framework within such models. In this respect, the contribution of the paper is to show how to deal with such heterogeneity and derive a quadratic welfare criterion as a function of aggregate variables only. The derivation applies to both perpetual-youth and DAMP models.

The third contribution is heuristic. The paper shows that the discontinuity of asset market participation implies an additional dynamic distortion in the model, which results in an additional term in the welfare criterion: while the social planner allocation implies that the cross-sectional consumption distribution is invariant to aggregate shocks, in the decentralized allocation it endogenously responds to any shock inducing fluctuations in aggregate financial wealth. Hence, financial stability appears in the welfare-based loss function and becomes an explicit additional target for welfare-maximizing policy makers, besides inflation and output stability. Finally, in this economy an endogenous trade-off arises between real and financial stabilization, even in the absence of cost-push or financial shocks. The ultimate implication is that price stability is no longer necessarily optimal: given the quadratic form of the welfare criterion, some fluctuations in output and inflation may be optimal as long as they
reduce financial instability. Ignoring the heterogeneity of asset-market participants potentially leads monetary policy to incur substantially higher welfare losses, under both discretion and full commitment.

References


35. A more elaborate source of agents heterogeneity may affect this result and the implied volatilities of relevant variables, like the cross-sectional consumption dispersion. Indeed, since the meaningful heterogeneity here comes from the perpetual-youth block of the economy, its structure is very stylized and downplays the effects that business cycle risk – and therefore fluctuations in the output gap – may have on the cross-sectional consumption dispersion.


