Ambiguity, Monetary Policy and Trend Inflation

Riccardo M. Masolo† and Francesca Monti‡

† Bank of England and Centre for Macroeconomics
‡ King’s Business School and Centre for Macroeconomics

Disclaimer: Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England, of any of its policy committees or to state Bank of England policy
Motivation

Three inflation-related concepts:

1. Target;
2. Trend;
3. Long-Run Expectations.

▶ In macro models they tend to be treated as one and the same variable.
▶ Empirical evidence (Chan, Clark and Koop, 2017) shows they do not coincide.

⇓

▶ It is important to model them separately;
▶ and to study the policy implications.
Target vs Expectations

Special questions from the 2007Q4 SPF Survey.

- Do you think the Fed follows a numerical target for long-run inflation? If so, what value?
- Respondents also provided their expectations for inflation over the next 10 years.

Table: 2007 Q4 SPF Special Survey

<table>
<thead>
<tr>
<th></th>
<th>Targeters</th>
<th>Non-Targeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of Responders</td>
<td>48</td>
<td>46</td>
</tr>
<tr>
<td>Average Target</td>
<td>1.74</td>
<td>n.a.</td>
</tr>
<tr>
<td>10-yr PCE Inflation Expectation</td>
<td>2.12</td>
<td>2.25</td>
</tr>
<tr>
<td>Short-rate Dispersion</td>
<td>.49</td>
<td>.61</td>
</tr>
</tbody>
</table>
TREND vs EXPECTATIONS

▶ Inflation exhibits low-frequency trend:
  - drives dynamics (Stock and Watson, 2007);
  - and persistence (Ascari and Sbordone, 2014).

▶ Most models ignore it.
  - Some treat variations in trend as a target shock, e.g. Del Negro, Giannoni and Schorfheide (2015);
  - or as an exogenous construct, e.g. Ascari and Sbordone (2014).

▶ Long-run inflation expectations are central to monetary policy.
  - Carvalho et al. (2017) capture their dynamics, but do not distinguish them from trend.

▶ Chan, Clark and Koop (2017): trend inflation and long-run inflation expectations have a time-varying relation and should not simply be equated.
The Cleveland Fed’s estimate is the solid red line, the SPF 10Y ahead inflation expectations is the dashed-dotted line and the stars are the Blue Chip measure of 5-10 years ahead inflation expectations. The black line is trend inflation estimated with the TVP-BVAR.
What We Do

- We introduce ambiguity about the behavior of the policymaker in a simple New-Keynesian model:
  - agents entertain as possible not one but a set of beliefs,
  - are unable to assign probabilities to them,
  - and they are averse to ambiguity

- Ambiguity gives rise to wedges between target, trend and long-run expectations,

- which have implication for monetary policy,

- and which we bring to the data.
Main Results

- We can rationalize:
  - the observed difference between long-run inflation expectations and trend inflation;
  - the evolution of trend inflation and inflation expectations since the early 1980s;
  - the below-target trend inflation since 2009/2010.

- We characterize optimal monetary policy rules in the presence of ambiguity.
LITERATURE

1. Optimal MP design in small NK models. An incomplete list includes:
   - King and Wolman (1996);
   - Yun (2005);
   - Schmitt-Grohé and Uribe (2007);

2. Ambiguity in business cycle models (Theory):
   - Ilut and Schneider (2014)
   - Bhandari, Borovička, and Ho (2019);

3. Ambiguity and Monetary Policy:
   - Cogley, Colacito, Hansen and Sargent (2008);
   - Adams and Woodford (2012);
   - Benigno and Paciello (2014);
The Model: Key Features

▶ Standard small New-Keynesian model (similar to Yun, 2005 and Galí, 2008):
  - No capital;
  - Sticky prices (Calvo, 1983);
  - Competitive labor market.

▶ The private sector is not fully confident about its understanding of the monetary policy rule

▶ Agents are averse to ambiguity.
Household’s Problem

The household maximizes:

\[
U_t(\vec{C}; s^t) = \min_{\mu_t \in [\underline{\mu}_t, \bar{\mu}_t]} \mathbb{E}^{\mu} \left[ u(\vec{C}_t) + \beta U_{t+1}(\vec{C}; s_t, s_{t+1}) \right]
\]  

subject to:

\[
P_t C_t + B_{t+1} = R_{t-1} B_t + W_t N_t + T_t.
\]

The felicity is given by:

\[
u(\vec{C}_t) = \log[C_t] - \frac{N_t^{1+\psi}}{1 + \psi}.
\]
Household’s First-Order Conditions

\[
\frac{1}{C_t} = \mathbb{E}^\mu_t \left[ \frac{\beta R_t}{C_{t+1} \Pi_{t+1}} \right]
\]

\[
N^\sigma_t C_t = \frac{W_t}{P_t}
\]

\[
\mathbb{E}^\mu_t \left[ \frac{\beta R_t}{C_{t+1} \Pi_{t+1}} \right] \equiv \mathbb{E}_t \left[ \frac{\beta \tilde{R}_t}{C_{t+1} \Pi_{t+1}} \right]
\]

Hence the intertemporal Euler equation becomes:

\[
\frac{1}{C_t} = \mathbb{E}_t \left[ \frac{\beta \tilde{R}_t}{C_{t+1} \Pi_{t+1}} \right]
\]
Monetary Policy

The Central Bank follows a very simple reaction function:

\[ R_t = \left( R^n_t e^{\zeta_{t+1}} \right) (\Pi_t)^{\phi}, \]  

(2)

where \( R^n_t = \mathbb{E}_t \frac{A_{t+1}}{A_t} \) is the natural rate of interest and \( \zeta_{t+1} \) is an autoregressive disturbance/mismeasurement in the natural rate.

If it wasn’t for \( \zeta_{t+1} \) the rule would be:

- **dynamically optimal**: stabilizes inflation and the output gap;
- **together with the subsidy**, **statically optimal**: it implements first best.

We aim at isolating what can go wrong despite an otherwise optimal policy regime.
Monetary Policy: Summary

The households are uncertain about the conduct of monetary policy

\[ R_t = \left( R^n_t e^{\zeta_{t+1}} \right) (\Pi_t)\hat{\phi}, \]

where the perceived law of motion of \( \zeta_{t+1} \) is:

\[ \zeta_{t+1} = \rho^{\zeta} \zeta_t + u^{\zeta}_{t+1} + \mu_t, \quad 0 < \rho^{\zeta} < 1 \]

\[ u^{\zeta}_{t+1} \sim \mathcal{N}(0, \sigma_u), \]
\[ \mu_t \in [\underline{\mu}_t, \bar{\mu}_t]. \]

→ Households base their consumption-savings decision on a distorted belief of the prevailing interest rate, which we refer to as \( \tilde{R}_t \).
FIRM’S PROBLEM

- Firms operate a linear production function: $Y_t(i) = A_t N_t$;
- receive a cost subsidy $\tau = 1/\epsilon$;
- maximize expected profits subject to Calvo frictions:

$$\max_{P_t} E_t \left[ \sum_{s=0}^{\infty} \theta_s Q_{t+s} \left( \left( \frac{P_t^*}{P_{t+s}} \right)^{1-\epsilon} Y_{t+s} - \Psi \left( \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right) \right) \right]$$

Which results in the following first-order conditions:

$$\frac{P_t^*(i)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \beta^j \theta^j MC_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{-\epsilon}}{E_t \sum_{j=0}^{\infty} \beta^j \theta^j \left( \frac{P_t}{P_{t+j}} \right)^{1-\epsilon}}$$

$$\frac{P_t^*(i)}{P_t} = \frac{1 - \theta \Pi_t^{\epsilon-1}}{1 - \theta} \left( \frac{1}{1 - \epsilon} \right).$$

Government and Market Clearing
Analysis of Model Properties

- We are concerned with long-run expectations,

- and put a premium on analytical solutions.

- We follow the trend-inflation tradition (Ascari and Sbordone, 2014) and focus on steady states and an anticipated-utility interpretation of how changes in the inflation trend affect the economy.

- Yet it is also possible to characterize the worst-case numerically solving the model globally. 

  Global Solution
**Steady-State Inflation**

**Proposition**

If agents perceive the disturbance $\zeta_{t+1}$ to have non-zero mean, inflation in steady state, relative to target, takes value:

$$\Pi(\mu, \omega) = e^{-\frac{\zeta}{\phi-1}}.$$ (6)

where $\zeta = \frac{\mu}{1-\rho^\zeta}$.

As a result, for any $\omega \in \Omega$, $\mu > 0 \Rightarrow \Pi(\mu, \omega) < \Pi(0, \omega) = 1$, while the opposite is true for $\mu < 0$.

where $\omega = [\beta, \epsilon, \theta, \phi, \rho^\zeta, \rho^a, \psi]$, and $\Omega$ is the set of admissible parameter values:

$$\Omega = \left\{ \omega : (\beta \in (0, 1), \epsilon \in (1, \infty), \theta \in (0, 1), \phi \in (1, \infty), \rho^\zeta \in (0, 1), \rho^a \in (0, 1), \psi \in [0, \infty) \right\} \cap \left( \rho^\zeta + \frac{\epsilon \mu}{\log(\theta)(\phi - 1)} < 1 \right).$$ (7)
Steady-State Analysis

- The interest rate used for decision-making purposes is not the one set by the Central Bank.

↓

- Inflation will not hit the first-best level.

- Price dispersion will emerge \(\Rightarrow\) Welfare falls (larger wedge between hours and consumption).

- *This effect arises both when inflation is inefficiently high or low.*

- Not obvious a priori which one is worse (unlike the TFP shock in Ilut and Schneider, 2014).
Steady-State Welfare as a Function of $\mu$
Steady-State Welfare as a Function of $\mu$

$$\pi^W = \pi^* - \frac{\mu}{(1 - \rho^\zeta)(\phi - 1)}$$
Worst-Case Steady State

Proposition

For any $\omega \in \Omega$, steady state welfare $\nabla(\mu, \omega)$ is continuously differentiable around $\mu = 0$ and:

i. attains a maximum at $\mu = 0$,

ii. is strictly concave in $\mu$,

iii. if the bounds are symmetric around zero $(\mu = -\bar{\mu})$, for $\beta$ sufficiently close to one, attains its minimum on $[-\bar{\mu}, \bar{\mu}]$ at $\mu = -\bar{\mu}$.

Key exception:

- Near the ZLB, ambiguity is (almost necessarily) one-sided.
- The symmetry condition is not verified.
- The worst-case corresponds to $\mu = \bar{\mu} > 0$ and long-run inflation expectations below the target level.
Worst Case: Economic Intuition

- A firm which does not re-optimize its price has its demand affected by inflation.

- When inflation is ”too low” the relative price increases: in the limit the firm’s demand will go to zero.

- When inflation is ”too high” the relative price decreases: firm’s demand is very high while its revenues plummet (in real terms).

- The latter case is worse.
Two Steady-State Concepts

1. Agents expect $\zeta_{t+1}$ to be negative on average (this drives their expectations).

2. In reality $\zeta_{t+1}$ has zero mean.

3. This is a negative news shock that does not materialize.

4. Endogenous variables will, in general, differ from their worst-case steady state value.

5. We refer to their average value as their ergodic steady state (what an econometrician would observe in the data).
Proposition

The ergodic steady state for inflation, in deviation from target, can be expressed in logs as:

\[
\overline{\pi} = \pi^W - \frac{\mu \lambda_{\pi\zeta} (\mu, \omega)}{1 - \rho^\zeta} = -\frac{\mu}{1 - \rho^\zeta} \left( \frac{1}{\phi - 1} + \lambda_{\pi\zeta} (\mu, \omega) \right)
\]  

(8)

\[
\lambda_{\pi\zeta} (\mu, \omega) \equiv -\frac{\kappa_0 \rho^\zeta}{(1 - \rho^\zeta) \left( 1 + \kappa_0 \frac{\phi - \rho^\zeta}{1 - \rho^\zeta} - \rho^\zeta \left( \kappa_2 + \rho^\zeta \frac{\kappa_1 \kappa_5}{1 - \rho^\zeta \kappa_6} \right) \right)}
\]

(9)

where \( \pi^W = \log \Pi^W (\mu, \bar{\mu}, \omega) \), \( \lambda_{\pi\zeta} (\mu, \omega) \) is the coefficient governing the equilibrium response of inflation to \( \zeta_t \), and the \( \kappa \)'s are functions of \( (\mu, \omega) \) which represent the coefficients in the log-linearized set of equilibrium condition.
Worst-case, Ergodic and Target levels of inflation

How do the three concepts of inflation relate to each other?

**Proposition**

For small $\mu$, for any $\omega \in \Omega^0$:

1. $-\frac{1}{\phi-1} < \lambda_{\pi \zeta}(\mu, \omega) < 0$,
2. $\pi^W$ and $\pi$ are both decreasing in $\mu$,
3. when the worst case corresponds to $\mu = \underline{\mu} < 0$ ($\mu = \bar{\mu} > 0$),
   $0 < \bar{\pi} < \pi^W$ ($0 > \bar{\pi} > \pi^W$).
Worst-case steady state inflation, ergodic steady state inflation, and inflation target (black dashed), as a function of $\mu$ (measured in basis points of annualized rate).
**Optimal Monetary Policy: Overview**

- Absent ambiguity, a monetary rules of the form:
  \[ R_t = R^n_t \Pi_t^\phi, \text{ for any } \phi > 1, \]
  is optimal.

- Under ambiguity, a policymaker (to the extent that she cannot reduce ambiguity further) can do better by following a rule in which:
  1. the ”intercept” is given by \( R_t^* = R^n_t e^\delta, \ \delta \neq 0; \)
  2. the ”slope” is chosen optimally.
Proposition

For any $\omega \in \Omega$, a small $\mu > 0$, $\mu = -\bar{\mu}$ and $\underline{\phi} \leq \phi \leq \bar{\phi}$, the following rule is steady-state optimal in its class:

$$R_t = R^*_t \Pi_t^{\bar{\phi}}$$

where $R^*_t = R^*_t e^{\delta^*(\bar{\mu}, \bar{\phi}; \cdot)}$ and $0 < \delta^*(\bar{\mu}, \bar{\phi}; \omega) < \frac{-\mu}{1 - \rho \zeta}$ is implicitly defined by $\nabla \left( -\bar{\mu} + \delta^*(\bar{\mu}, \bar{\phi}; \cdot), \cdot \right) = \nabla \left( \bar{\mu} + \delta^*(\bar{\mu}, \bar{\phi}; \cdot), \cdot \right)$.

A simple corollary shows that, near the ZLB, $0 > \delta^* > -\frac{\mu}{1 - \rho \zeta}$, while the optimal $\phi$ remains the same.
Optimal Policy

- Lack of credibility/ambiguity makes the responsiveness to inflation deviations from target critical;

- when worst-case expectations exceed the target, policy should be systematically more *hawkish* than it should in the absence of ambiguity (Volcker ”excessive” tightening in the early 80’s);

- near the ZLB, it should be more *dovish* (lower for longer).

- None of these rule can attain first best: to the extent possible it is preferable to reduce ambiguity.
**Robustness**

1. Constant probability of price change.
   - robust to state-depended pricing (Dotsey, King, and Wolman, 1999) as well as Rotemberg (1982) and Taylor (1979).

2. Zero net-supply of bonds $\Rightarrow$ pessimistic expectations do not affect wealth.

3. Representative-agent neglects possible *Fisherian* effects.

4. Firms assumed to be as ambiguity-averse as households.
We extend our model to have Borrowers and Savers:

1. agents differ in their discount factor (Iacoviello, 2005; Bilbiie, Monacelli, and Perotti, 2013): \( 1 > \beta^S > \beta^B > 0 \);

2. they are subject to a borrowing constraint \( D_t \geq -\overline{D} \), which is binding for Borrowers in equilibrium.

3. The government maintains a constant level of government debt (levying lump-sum taxes on Savers to pay for interests and the firm subsidy).
Inefficiency vs Fisherian Effect
**Inefficiency vs Fisherian Effect**

- The *Inefficiency* effect dominates when:
  - the level of ambiguity is sufficiently high;
  - rates are low ($\beta^S \to 1$);
  - demand is very elastic (high $\epsilon$) and/or the degree of price stickiness is high (high $\theta$).

  The worst-case is identical for both agents in the model and equal to that from the representative-agent model.

- When the *Fisherian* effect prevails:
  - Borrowers’ worst-case expectations will correspond to lower inflation;
  - Savers’ worst-case expectations will correspond to higher inflation.

  We need solve for the ergodic steady state under heterogeneous worst-case expectations (Ilut, Kryvenko and Schneider, 2019).
Worst-Case Inflation Expectations under the Fisherian Effect

![Graph showing worst-case inflation expectations under the Fisherian effect. The graph plots Ergodic π, Savers Expectations, Borrowers Expectations, and Mean Expectations against \( \bar{\mu} \) (annualized b.p.). The graph includes a note: Pessimistic Exp’ns.](unnamed.png)
Implications of the Borrower-Saver Model

Our key findings do not depend on:

- the representative-agent assumption;
- the zero-net supply of bonds;
- the assumption that firms have the same inflation expectations as their owners.
Testable implications

When the level of ambiguity that minimizes welfare is $\mu (\overline{\mu})$:

- both the worst-case steady-state inflation and the ergodic steady state inflation are above (below) target;
- statistical measures of trend inflation should lie between long-run inflation expectations and the target;
- as the degree of ambiguity falls, all measures should tend to converge to the inflation target.
Bringing our model to the data

- Agents are ambiguous about the monetary policy rule.
  \( \rightarrow \) match \([\mu, \bar{\mu}]\) with disagreement in the Blue Chip nowcasts of FFR

- They will make their decisions \textit{as if} worst case scenario materializes.
  \( \rightarrow \) match \(\pi^W\) with long-run inflation expectations (Cleveland Fed, Blue Chip)

- Worst case does not materialize \( \rightarrow \) ergodic steady state
  \( \rightarrow \) match \(\bar{\pi}\) with statistical measures of trend inflation (TVP-BVAR Cogley-Sbordone, 2008)
DISAGREEMENT OF BLUE CHIP NOWCASTS OF FFR

Masolo & Monti  
AMBIGUITY, MONETARY POLICY AND TREND INFLATION  
Teaching Slides
Putting Symmetry to the Test

- Symmetric bounds imply $\bar{\pi} \geq \pi^*$
- The ZLB is an obvious candidate for asymmetric bounds

![Graph showing skewness statistic and alpha = 1% over time from 1985 to 2015.](image)
Estimation

We use a simple minimum-distance estimation, we estimate $\omega$ by solving the following minimization:

$$\min_{\omega \in \tilde{\Omega}_0} \sum_{t=1}^{T} m_t (\omega, z_t)' W m_t (\omega, z_t)$$

Where

$$m_t (\omega, z_t) = \begin{bmatrix} z_t^{\pi_e} - \left( \pi^* - \frac{z_t^{\mu}}{(1-\rho^\zeta)(\phi-1)} \right) \\ z_t^{trend} - \left( \pi^* - \frac{z_t^{\mu}}{1-\rho^\zeta} \left( \frac{1}{\phi-1} + \lambda \pi\zeta (z_t^{\mu}, \omega) \right) \right) \end{bmatrix}, \quad (10)$$

and $W$ the identity matrix.

We calibrate $\beta = .995, \epsilon = 11, \phi = 1.5$. 
### Estimation Results

<table>
<thead>
<tr>
<th>ClevFed 10Y</th>
<th>BC 5-10Y</th>
<th>SPF 10Y</th>
<th>Mich 5-10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.5275</td>
<td>0.5834</td>
<td>0.5231</td>
</tr>
<tr>
<td></td>
<td>(0.468, 0.627)</td>
<td>(0.390, 0.677)</td>
<td>(0.218, 0.715)</td>
</tr>
<tr>
<td>$\rho^\xi$</td>
<td>0.7335</td>
<td>0.7395</td>
<td>0.6962</td>
</tr>
<tr>
<td></td>
<td>(0.704, 0.778)</td>
<td>(0.719, 0.811)</td>
<td>(0.662, 0.814)</td>
</tr>
<tr>
<td></td>
<td>129</td>
<td>65</td>
<td>97</td>
</tr>
</tbody>
</table>

**Table:** Estimates of $\theta$ and $\rho^\xi$ obtained using different measures of long-run inflation expectations. We indicate in parentheses the 95% confidence intervals obtained by bootstrapping, using the moving block method proposed by Künsch (1989) for dependent data. The length of the blocks is of 4 quarters, but we experimented with different block length and found that the results are robust to the choice of block length.

†: The series comprises biannual observations over 1983Q4-198Q4 and quarterly observations over 1986Q1-1987Q4 and 1990Q2-2015Q4, for a total of 115 observations. Data for the remaining 14 quarters has been generated by interpolation.
**Inflation as a function of ambiguity**

10-years inflation expectations Cleveland Fed, Measure of trend inflation from a TVP-VAR
Conclusions

▶ Changes in ambiguity (transparency) can explain why long-run inflation expectations differ from statistical measures of trend and they both differ from the trend.

▶ We can rationalize the convergence of expectations and measures of trend towards the 2 percent target over the 80s and 90s,

▶ and their fall below the target after the Great Recession.

▶ The optimal policy stance depends on the degree of ambiguity and the level of trend inflation.
CPI inflation trend inflation implied by a TVP-BVAR using GDP deflator (blue), CPI (bold black line), PCE deflator (red). The dotted lines indicate the 90% confidence bands for the trend inflation obtained using CPI as a measure on inflation.
Mainly, two alternative preferences specifications used for representing ambiguity aversion in macro:

   - Multiple priors utility is not smooth when belief sets differ in means.
   - Effects of ambiguity show up in a first order approximation
     Ilut and Schneider (2014)

   - Fear of misspecification: statistical perturbation around an approximating model.
   - Smooth utility function
**Government and Market Clearing**

The government taxes to finance the subsidy. We lump the profits together with the tax, which results in the following:

\[
T_t = P_t \left( -\tau \frac{W_t}{P_t} N_t + Y_t \left( 1 - (1 - \tau) \frac{W_t \Delta_t}{P_t A_t} \right) \right) = P_t Y_t \left( 1 - \frac{W_t \Delta_t}{P_t A_t} \right)
\]

where \( \Delta_t \) is the price dispersion term, derived from the market clearing condition for the labor market:

\[
N_t = \int_0^1 N_t(i) \, di = \int_0^1 \frac{Y_t(i)}{A_t} \, di = \frac{Y_t}{A_t} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} \, di,
\]

where we define \( \Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} \, di \) (Yun, 2005), which evolves as:

\[
\Delta_t = \theta \Pi_t^\epsilon \Delta_{t-1} + (1 - \theta) \left( \frac{1 - \theta \Pi_t^{\epsilon-1}}{1 - \theta} \right)^{\frac{\epsilon}{\epsilon-1}}.
\]
Consider the linearized version of the policy rule:

\[ i_t = r^n_t + \phi \pi_t + \zeta_{t+1}, \quad \zeta_{t+1} = \rho \zeta_t + \mu + u_{t+1}^\zeta, \quad u_{t+1}^\zeta \sim \mathcal{N}(0, \sigma_u), \]

Define \( x_t = r^n_t + \phi \pi_t \) and consider the probability attached to an event:

\[ Pr(i_t < 2\%) = Pr(\zeta_{t+1} < 2\% - x_t) = \Phi \left( \frac{2\% - x_t - \rho \zeta_t - \mu}{\sigma_u} \right). \]

We assume that agents:

- know \( \sigma_u \);
- do not attach any probability to \( \mu \in [\mu, \bar{\mu}] \);
- so they evaluate \( Pr(i_t < 2\%) \) using the level of \( \mu \in [\mu, \bar{\mu}] \) that makes them worse off.